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TENSOR TRIANGULATED CATEGORY TO A CYCLES DUALITY IN THE QUANTUM VERSION OF MOTIVIC COHOMOLOGY

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Abstract. We consider the tensor structure of triangulated categories in derived categories of étale sheaves with transfers. The total tensor product on the category PSL(k), is required to obtain the generalizations on derived categories using presheaves, contravariant and covariant functors on additive categories of the type $\mathbb{Z}(A)$, or $A^{\oplus}$, to determine the exactness of infinite sequences of cochain complexes and resolution of spectral sequences. Further, considering all referent to a triangulated category whose motivic cohomology is a hypercohomology from the category Sm_k, and the implications in the geometrical motives applied to a bundle of geometrical stacks in field theory, that is to say, to the context to the category DQFT, was established a lemma that incorporates a 2-simplicial decomposition of $\Delta^3 \times A^1$, in four triangular diagrams of derived categories from the category Sm_k, whose goal was evidence the tensor structure of DQFT. Now in this research, we consider a theorem that relates the hypercohomology groups obtained with the spectrum through the its singular homology taking components $\mathbb{Z}_{tr}(k)$, and the A^1- homotopy in the action of the symmetric group on the derived category $\mathrm{DM}_{Nis}^{eff,-}(k)$. This finally give us a crystallographic space-time model of simplicial type from the microscopic aspects that define it, and its re-interpretation in field theory under the dualities of the hypercohomology Nisnevich groups that are the vertices in the decomposition of the space $\Delta^3 \times A^1$, which are equivalent to the field waves, for example gravitational waves. Then is established an isomorphism of waves and particles in the context of the category $\mathrm{DM}_{gm}(k)$. The equivalence exists in this category.

1. Introduction

We remember that in before papers [1,2] we consider commutative diagrams constructed from the category PSL(k), is Abelian [3] and therefore has enough injectives and projectives that can be used to create the conditions for the invariant presheaves of homotopy required to realization of the commutative diagrams in A^{1-} homotopy of morphisms in the category Sm_k, as the corresponding diagrams of A^{1-} morphisms in the category $C_*\mathbb{Z}_{tr}(X \times A^1)$ (Figure 1),

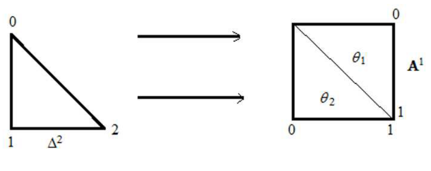

Figure 1. 2-Simplicial decomposition of $\Delta^2 \times A^1$.

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which is a 2-Simplicial decomposition of $\Delta^2 \times A^1$. Or the case of consider Δ^3, we have the correspondence(Figure 2):

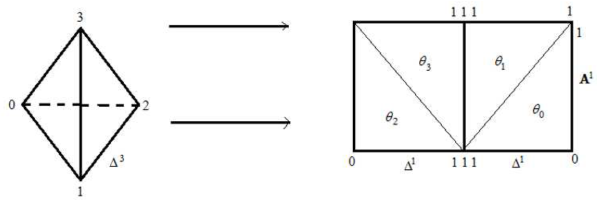

Figure 2. 2-Simplicial decomposition of $\Delta^3 \times A^1$.

which represents a 2-Simplicial decomposition of $\Delta^3 \times A^1$. This last case, was used to obtain a general diagram that was induced to the category DQFT, from a scheme of associated motives to scheme X, (which is the class $m(X)$ of $C_*\mathbb{Z}_{tr}(X)$, which is clearly modulus A^1-homotopy in an approximate triangulated category $\mathrm{DM}_{Nis}^{eff,-}(k, R)$[1], constructed from the derived category of PSL(k). Remember that the hypercohomology determined in [4, 5] to solutions of a big class of field equations corresponding to the representation of the cosmic Galois group[2], establish that in quantum field theory also can be established that the motives are objects whose triangulated category of geometrical motives are in the category $\mathrm{DM}_{gm}(k, R)$, or written simply as $\mathrm{DM}_{gm}(k)$.

The following corollary to homotopy invariant presheaves [6, 7] and deduced from the fact that for every smooth scheme X, exists a natural homomorphism (is to say a homotopy) which explained a diagram that belongs to the correspondence planted in (Figure 1), or as factor of the correspondence planted in the (Figure 2). Likewise we have:

Corollary 1. *$C_*\mathbb{Z}_{tr}(X \times A^1) \to C_*\mathbb{Z}_{tr}(X)$, is a chain homotopy equivalence.*

Then in the motives context and after of demonstrate the equivalences (in A^1- homotopy) of the correspondences morphisms of injectives and projections, we can to have the motives scheme equivalence $m(X) \cong m(X \times A^1)$, for all X, which helps to establish in a general way that any A^1- homotopy equivalence $X \to Y$, induces an isomorphism $m(X) \cong m(Y)$, considering inverses.

2. Derived triangulated categories with structure by pre-sheaves $\otimes^L$ and $\otimes^{tr}_{L,ét}$

The tensor product of the derived category of bounded above complexes of étale sheaves of $R-$ modules $\otimes^{tr}_{L,ét}$, preserves quasi-isomorphisms. Also the category of bounded above complexes of étale sheaves of $R-$modules with transfers is a tensor triangulated category[8,9].

Particularly, and considering a motives algebra into the derived category of étale sheaves of a $\mathbb{Z}/m-$ module with transfers, we have the operation

$$m \to m(1) = m \otimes^{tr}_{L,ét} \mathbb{Z}/M(1) \tag{1}$$

is inversible. Then $\forall E, F$, étale sheaves we have

[1]The category is endowed of total tensor product inherited from the total tensor product of PSL(k).
[2]$K_{2n-1}(K) \otimes \mathbb{Q} = H_\bullet(GL(n, k))$, is the linear group of entries in k.

$$E \otimes^{tr}_{L,ét} F \to E' \otimes^{tr}_{L,ét} F \tag{2}$$

is quasi-isomorphism for F. Now if F, is a locally complete étale sheaf of $R-$modules then $E' \otimes^{tr}_{L,ét} F, \to E \otimes^{tr}_{L,ét} F$, is quasi-isomorphism for every étale sheaf with transfers E. But in the aspects of tensor products we have $\otimes \cong \otimes^L$, in $\wp$, and considering the a natural mapping of presheaves established by $\lambda : h_X \otimes^{tr}_{ét} h_Y \to h_{X \otimes^{tr}_{ét} Y}$,for every $h_{X_i} = R(X_i)$, considering the right exactness of $\otimes_R$, and $\otimes^{tr}_{ét}$, and being E, F, are bounded above complexes of locally constant étale sheaves of $R-$module the $E \otimes^{tr}_{L,ét} F \to E \otimes^{\mathbb{L}}_R F$, is also a quasi-isomorphism.

Similarly as has been with the étale sheaves, a presheaf with functors F, is a Nisnevich sheaf with transfers if its underlying presheaf is a Nisnevich sheaf on the category Sm/k. Consequently every étale sheaf with transfers is a Nisnevich sheaf with transfers. In the motive context with $\mathbb{Q}$-coefficients with transfers we can enunciate the following result.

Lemma 1. *Let F, be a Zariski sheaf of $\mathbb{Q}-$ étale sheaf with transfers. Then F, is also an étale sheaf with transfers.*

Proof. 1,7 □

Then we can deduce considering the theorem that characterizes the Nisnevich sheaves [6-8] whose category is the space $Sh_{Nis}(Cor_k)$, and the lemma 1, the following corollary.

Corollary 2. *If F, is a presheaf of $\mathbb{Q}-$modules with transfers then $F_{Nis} = F_{ét}$.*

Likewise, the construction of a derived category as $\mathrm{DM}^{eff.-}_{Nis}(k, R)$, is analogous to the construction of $\mathrm{DM}^{eff.-}_{ét}(k, R)$. If k, admits regularizations of singularities then $\mathrm{DM}^{eff.-}_{ét}(k, R)$, permit us to extend motivic cohomology to all schemes of finite type as a cdh, hypercohomology group.

Remember that $\mathbb{Q} \subseteq R$, which showed us that $\mathrm{DM}^{eff.-}_{Nis}(k, R)$, and $\mathrm{DM}^{eff.-}_{ét}(k, R)$, are equivalent [1, 7]. Then $\mathrm{D}^- = D^-(Sh_{ét}(Cor_k, R))$, is a derived category which is a tensor triangulated category. The same is applicable in the Nisnevich topology for derived category $\mathrm{D}^-(Sh_{Nis}(Cor_k, R))$.

Then, $\forall C, D \in \wp$, and thus in $\mathrm{Ch}^- R(\mathcal{A})$, we have:

$$C \otimes^{tr}_{L,Nis} D \cong (C \otimes^{tr}_L D)_{Nis'}. \tag{3}$$

Particularly the derived category D^-, of bounded above complexes of Nisnevich sheaves with transfers is a tensor triangulated category under the tensor product $\otimes^{tr}_{L,Nis}$. Then by the proposition [7] that affirms that $h_X = R_{tr}(X)$ is projective if

$$R_{tr}(X) \otimes^{tr} R_{tr}(Y) = R_{tr}(X \times Y). \tag{4}$$

Then we have in the motives context

$$m(X) \otimes^{tr}_{L,Nis} m(Y) = m(X \times Y). \tag{5}$$

Then can be defined the category $DM^{eff}_{gm}(k, R)$, as the thick subcategory of $DM^{eff.-}_{Nis}(k, R)$, generated by all motives $m(X)$, where X is smooth over k. The objects in $DM^{eff}_{gm}(k, R)$, are the effective geometric motives, which will be the useful objects required in our motivic cohomology that we want establish and that we obtain for resolution of the decomposing of $X \times A^1$ in A^1-homotopy of morphisms in the category Sm_k.

This will be the beginning of the next section to a category DQFT, already characterized and studied in [1,4].

3. Result

Many studies realized and published in [5] and the motivic cohomology treatment given in [6-8, 10] as the embedding theorem in $\mathrm{DM}_{\acute{e}t}^{eff}(k)$, can be considered the following triangule:

$$\begin{array}{c} Sm_k \to DM_{\acute{e}t}^{eff}(k) \\ m \searrow \qquad \downarrow Id, \\ DM_{\acute{e}t}^{eff}(k) \end{array} \tag{6}$$

with implications in the geometrical motives applied to the bundle of geometrical stacks in mathematical physics, as has been studied and showed in [6, 8, 11]. An important theorem was obtained in [4, 5], where considering $\mathbb{M}$, as space-time modeled as a complex Riemannian manifold with singularities was obtained the following tensor triangulated diagram, which is true and commutative:

$$\begin{array}{c} DQFT \\ i \swarrow \qquad \searrow F, \\ DM_{gm}(\mathbb{Q}) \to DM(\mathcal{D}_Y) \end{array} \tag{7}$$

Proof. 8. □

The category $\mathrm{DM}_{gm}^{eff}(k, R)$, as tensor triangulated category has a tensor product of its motives describe as $m(X) \otimes m(Y) = m(X \times Y)$. An important note is consider that the triangulated category of geometrical motives $\mathrm{DM}_{gm}(k, R)$, is defined formally inverting the functor of the Tate objects[3] which are objects of a motivic category. In before researches was considered a tensor triangulated category to a quantum version of motivic cohomology on étale sheaves, from Δ^3- simplicial that shows the A^1- homotopy in an approximate triangulated category $DM_{Niss}^{eff,-}(k, R)$, which for every Nisnevich sheaf with transfers, that is each one an every étale sheaf with transfers, is a category $DM_{\acute{e}t}^{eff,-}(k, R)$. The Nisnevich detail in the derived category is due to the importance in motivic homotopy theory of that the objects of interests are "spaces", which are simplicial sheaves of sets on the big Nisnevich site that is the category Sm/k. In reality we consider two topologies for aspects of localization and covering. We have the following commutative diagram in the geometrical motives context that are useful to link the derived category DQFT.

Lemma 2. *The following diagram is commutative*

$$\begin{array}{c} Sm_k \xrightarrow{i'} \mathrm{DM}_{gm}^{eff}(k) \xrightarrow{\sigma} \mathrm{DM}_{gm}(k) \xleftarrow{i} \mathrm{DQFT} \\ m \searrow \quad \updownarrow \mathrm{Id} \quad \sigma \swarrow \quad \updownarrow\cong \quad \swarrow F, \\ \mathrm{DM}_{gm}^{eff}(k) \xrightarrow{\cong} \mathrm{DM}(\mathcal{D}_Y) \end{array} \tag{8}$$

Proof. 13. □

[3]The Tate motives or mixed Tate motives (the mixed Tate motives are the iterated extensions of the pure Tate motives, thus are same by nature) are central objects in the study of cohomology groups of algebraic varieties and their arithmetic invariants. These have a crucial role in several problems and questions related with the algebraic K-theory, hyperbolic geometry, and particle physics among others.

In the demonstration was considered QFT$\xrightarrow{i}$DM$_{gm}(k) \xrightarrow{\sigma}DM^{eff}_{gm}(k)$, which is zero (see lemma 21.9 [1,13]). And the high importance was consider the singular homology [14] to start Cor_k/A^1- homotopy.

An corollary of the diagram (6) will be the re-interpretation from the étale sheaves topology and simplicial decomposition of $\Delta^3 \times A^1$,for DQFT, considering their spectrum of its singular homology. This precisely will complete the research on the equivalence of co-cycles through its spectrum in a derived category of motives, will be consigned in a corollary.

From category of motives, we can consider the following proposition worked in [15]:

Proposition 1. If X, is any scheme of finite type over k, then

$$H_n^{sing}(X, R) \cong H_{n,0}(X, R). \tag{9}$$

Into the demonstration of (9) are considered the hyper-cohomology groups $\mathbb{H}^*_{Nis}(\text{Speck}, K) = H(K(\text{Speck}))$, which represent the spectrum of the corresponding singular homology. This spectrum can be a projective vector bundle used to work singularities. Then in a deep research realized in [2] was conjectured that oscillations and singularities can be the same in motivic cohomology through of certain duality.

For our goal will be very useful the following theorem.

Theorem 1. *(Projective Bundle Theorem). Let $p : \mathbb{P}(\varepsilon) \to X$, be a projective bundle associated to the vector bundle ε,of rank $n+1$. Then the canonical mapping*

$$\otimes^n_{i=0}\mathbb{Z}_{tr}(X)(i)[2i] \to \mathbb{Z}_{tr}(\mathbb{P}(\varepsilon)) \tag{10}$$

is an isomorphism in the category DM$^{eff}_{gm}(k)$, and p, is the projection onto the factor $\mathbb{Z}_{tr}(X)$.

Proof. 15. □

Then have the orthogonal composition[4]:

$$\mathbb{Z}_{tr}(\mathbb{P}^n_k = \otimes^n_{i=0}\mathbb{Z}(i), \tag{11}$$

of motivic complex of singularities.

Likewise, we can consider the following theorem proved and published in [12].

Theorem 2. (F. Bulnes) *$H^*(GL(n,k))$ has the decomposing in components $H^i(X)$, that are hyper-cohomology groups corresponding to solutions as* **H**$-$ *states in Vec$_{\mathbb{C}}$, for field equations $dda = 0$.*

Proof. 12. □

Into the demonstration of the theorem was proved that the oscillations of **H**$-$states are the solutions of a wide field equations class where these solutions are hyper-cohomology groups to superposition of **H**$-$states, considering the corresponding Hitchin base [11, 16, 17]. In field theory, considering duality, particle and wave are equivalent. Then oscillations or waves are singularities in the space-time too.*In our category of motives are undistinguishable.* In an after theorem proved in [2] was demonstrated in terms of singular cohomology that the group $H^*(SL(n,k))$, has a decomposing in components $\mathbb{Z}(i)[2i]$, that are hypercohomology groups to solutions as **H**$-$states in Vec$_{\mathbb{P}}$, and are solutions to a wide

[4] $\mathbb{Z}(n)$, is the motivic complex of manifold singularities whose dual as hypersurfaces in a manifold (that our case we want a complex Riemannian with singularities) is the projective space $\mathbb{P}^n$.

field equations class. In the demonstration is considered the Proposition 1, where is clear that:

$$H_n^{sing}(X,R)=\mathbb{H}^{-n}_{Nis}(\mathrm{Speck},C_*R_{tr}(X)). \tag{12}$$

Likewise results natural the following corollary considering the actions of the corresponding group on the complexes given.

Corollary 3. *The action of the symmetric group Σ_n, on $\mathbb{Z}(n)$, is $\mathbb{A}^1-$ homotopic to the trivial action. Hence it is trivial in the category $DM^{eff,-}_{Nis}(k)$, an on the motivic cohomology(hypercohomology)$\mathbb{H}^r(X,\mathbb{Z}(n))$.*

Proof. 15. □

Theorem 3. *We consider $H^*(SL(n,k))$. This has a decomposing in components $\mathbb{Z}(i)[2i]$,* [5] *that are hypercohomology groups to solutions as $\mathbf{H}-$states in Vec$_{\mathbb{P}}$, to field equations $dda=0$. on singularities.*

Proof. 12. □

Here the important point was use the diagram

$$\begin{array}{c} \mathrm{DM}_{gm}(k) \overset{i}{\leftarrow} \mathrm{DQFT} \\ \updownarrow\cong \qquad \swarrow F, \\ \mathrm{DM}(\mathcal{D}_Y) \end{array} \tag{13}$$

directly deduced from diagram (8), and use the Spec of the motivic hypercohomology $\mathbb{H}^*_{Nis}(\mathrm{Speck},k)$, considering the projective vector bundle.

Corollary 4. *The waves or oscillations are singularities in the microscopic space-time. In our category of motives are undistinguishable.*

Proof. In QFT-applications, the singular homology groups of $\Delta^3\times A^1$, for DQFT, are dual to the corresponding $\mathbf{H}-$states in Vec$_{\mathbb{C}}$, to the motivic cohomology. These are continuous in their spectrum $\mathbb{H}^{-n}_{Nis}(\mathrm{Speck},C_*R_{tr}(X))$. By the theorem 3.3 this spectrum can be a projective vector bundle $\mathbb{P}(\varepsilon)$, used to work singularities. Of fact there is a quiasi-isomorphism $M(\mathbb{P}^n)=C_*\mathbb{Z}_{tr}(\mathbb{P}^n)\to\mathbb{Z}\oplus\mathbb{Z}(1)[2]\oplus,...,\oplus\mathbb{Z}(n)[2n]$, that are Spec of corresponding Chow groups. In the demonstration of this theorem, oscillations or waves and singularities are the same considering motivic cohomology [2]. Of fact all motives are in a Tannakian category [8] generated for Tate motives.[6] □

In the case of a spinor representation the corresponding $\mathbf{H}-$states can be as spinor waves [18] which can be consigned in oscillations in the space-time to a microscopic context (for example all fermions, even some neutrino oscillations) deformation measured [19, 20] in $\mathcal{H}$. Now these waves states in field theory are motives in the category DTM(k)[4], which is the derived category D^b(TM)(k)), that is a full subcategory of the Tate category of motives TM(k),before mentioned. Then oscillations are singularities in the microscopic space-time. In our category of motives are undistinguishable. Then directly from the corresponding triangle of the diagram (8) all are geometrical motives of DM$_{gm}(k)$, and are in DM($\mathcal{D}_Y$), which is a category of the microscopic space-time.■

[5]These are the Spec of a corresponding Chow groups. We consider the following *Corollary* [15]. There is quasi-isomorphism

$$M(\mathbb{P}^n)=C_*\mathbb{Z}_{tr}(\mathbb{P}^n)\to\mathbb{Z}\oplus\mathbb{Z}(1)[2]\oplus,...,\oplus\mathbb{Z}(n)[2n].$$

[6]Let MT($\mathbb{Z}$), denote the category of mixed Tate motives unramified over $\mathbb{Z}$. It is a Tannakian category with Galois group Gal$_{\mathrm{MT}}$.The Tannakian category is obtained for the inverting of the objects $-\otimes\mathbb{Z}.(1)$.

4. Examples

The following examples sign different field theory objects where all these are mixed motives of the same derived category of motives, established in the demonstration of the corollary.

Example 1. *In the QFT-applications and considering the singular homology groups of* $\Delta^3 \times A^1$*, for DQFT, we can consider the duality that comes given by the isomorphism* $H^{sing}_{n-p}(\mathbb{G}_m^{\wedge n}, \mathbb{C}) \cong H^{n-p}(\mathbb{CP}^{m-1}, \mathbb{C})$*. This duality lives in the Tannakian category TM*(k) *[4].*

Example 2. *We consider the isomorphism* $Pic(\mathbb{P}) \cong H^2_{Nis}(\mathbb{P}, \mathbb{Z}(1)) \cong Hom_{D^-}(\mathbb{Z}_{tr}(\mathbb{P}), \mathbb{Z}(1)[2])$*.*[7] *By deformation theory, we cosider the Picard group of the Picard variety of a curve* C*, defined for* $\mathbb{M} = Pic(C)$*, where* $\mathbb{M}$*, is the microscopic space-time. In a physical context* $Pic(C)$*, represents a trace or curve of a particle in the symplectic geometry that can be characterized in a Hamiltonian manifold* $\mathcal{H}$*, as* $\mathbf{H}-$*states. In this case the particles(singularities)and* $\mathbf{H}-$*states are the same, in the case of* $Pic(\mathbb{P})$*, represents a wave at infinitum which is constructed or formed when more* $\mathbf{H}-$*states are being added indefinitely. Its Spec is* $T^\vee \mathrm{Bun}_{L_G}$*. The isomorphism(equivalence) comes given by the corresponding Penrose transform such that its kernel set has elements the fields* $\mathbf{h} \in \mathcal{H}$ *such that Isomd*$\mathbf{h} = 0$*, in the corresponding hyper-cohomology [10]. Likewise, in the Hamilton densities space [17] we have a superposition of* $\mathbf{H}-$*states, considering a Hitchin basis*[8]*[4]. Likewise, we have for a concrete case* $H^2(\mathrm{Bun}_G(\Sigma, {}^L G), \mathbb{C}) \cong \Omega^1[\mathbf{H}]$*, [12] (where the second differential operator is defined as* $d : \Omega^1[\mathbf{H}] \to C \times B$*) that for* $\mathbb{M} = Pic(C)$*, we obtain* $\mathbf{H}^\vee$*, which is its deformation or spectrum, whose general integral is the extended Hitchin base [17,18]* $\mathcal{H} = H^0(\omega_C) \oplus H^0(\omega_C^{\otimes 2}) \oplus ... \oplus H^0(\omega_C^{\otimes n})$*,* [9] *whose hypercohomology has an image in the category of spaces* $Vec_{\mathbb{C}}$*,[4].In the case of a spinor representation the corresponding* $\mathbf{H}-$*states can be as spinor waves, which can be consigned in oscillations in the space-time to a microscopic deformation measured [19, 20] in* $\mathcal{H}$*.*

Example 3. *Oscillations or waves of* $\mathbf{H}-$*states are produced when are realized rotations around of a vertex (this could be considered like a source in field theory)in presence of electromagnetic fields give a field torsion accompanied of gravitational waves as effect of the particles mass (that is to say, in presence of matter) (see the figure 1). To quantum gravity, might be natural to want obtain a spectrum in the dual* $\hat{T}Bun_G$*,* [10] *considering the triangle given in (8), whose geometrical motives will be stacks of holomorphic bundles.*

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[7]The category $D^- = D^-(Sh_{\acute{e}t}(Cor_k, R))$.

[8]This is possible if is had a projective bundle

[9]Each motive M, can be expressed as $\omega(M)$, which is direct sum of $\omega_n(M)$, which have images as the Grassmannians $\mathrm{Gr}^w_{-2n}(M)$. For realization of motives we have:

$$\mathrm{MT}_k \to \mathrm{GrVec}M(X) \mapsto H^*(X_k, \mathbb{Q}),$$

Each $\omega_C^{\otimes j}$, is a connection for $\mathbb{Z}-$ grade $\mathbb{Q}-$ vector spaces such that for all motives M, that are elements of a tensor derived category, is had that:

$$M \mapsto \omega(M) = \oplus_n \omega_n(M),$$

with

$$\omega_n(M) = \mathrm{Hom}(\mathbb{Q}(n), \mathrm{Gr}^w_{-2n}(M)).$$

[10]Dual image of the lines bundle which is divisor of holomorphic bundles. This is stack.

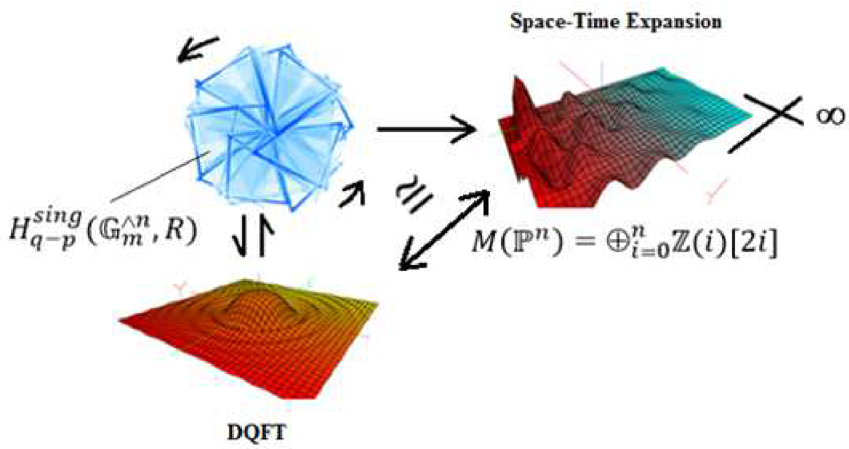

FIGURE 3. The triangle (35) in the demonstration of the theorem 3.3, [2] and the Chow ring of the hypersurface modeled considering the space-time expansion. We will consider this sequence as a sequence of coherent sheaves in $\mathbb{P}^n$. The cohomology of coherent sheaves is the same that the cohomology to étale sheaves.

5. Conclusions

The construction of a Universe model on simplicial framework, considering the quantum field theory of simplicial geometry establish morphism of homotopy commutative relations which can induce to a hypercohomology to the solution of some field equations and aspects of gravitation at least in a microscopic level. As example, we can consider the field theory as the established by the Schwinger-Dyson equation in 3-dimensional simplicial quantum gravity, established by new triangle relations and absence of Tachyons in a Liouville string field theory [19], whose $D-$modules are coherent $D-$modules and where could be contained in the derived category $\mathrm{DM}(\mathcal{D}_Y)$, or also in the diagrams of the Polyakov string theory [20], with marble Polyakov integrals as intertwining operators between strings and particles (sources as vertices). However the obtaining of a derived category of mixed motives where dualities between field theory objects of the derived category DQFT are defined by isomorphisms, even developed through stacks of the holomorphic bundles, establish in a advanced level the total equivalence of singularities and waves or oscillations, traying also the deformation theory meaning to corresponding co-cycles of their spectrum in each case. The corollary 3. 2., is a direct consequence of the theorems 3. 2, and 3.3, which are consequence of a previous study on geometrical motives categories to determine co-cycles as solutions in field theory [5], where previously also was demonstrated the commutativity of scheme of derived categories [12] to demonstrate the equivalence of solutions of certain field equations classes and obtain a same derived category of such co-cycles to both objects in QFT of field equations. Then the solutions of the all field equations classes are obtained in a hypercohomology. Likewise to give treatment to the singularities we use the simplicial geometry and its decomposition in triangulated diagrams of schemes belonging to the category Sm_k, and morphisms between schemes of the category Cor_k, all with the total tensor product on the category $\mathrm{PSL}(k)$, as example its component elements $\mathbb{Z}_{tr}(k)$, to obtain the generalizations on derived categories using sheaves (étale or Nisnevich) or pre-sheaves and contravariant and covariant functors on additive categories to define the exactness of infinite sequences and resolution their spectral sequences. The advantages from the tensor triangulated category to a quantum version considering a motivic cohomology on étale Sheaves is the respective factorization algebras in QFT, where is necessary consider the combined observation measures from many components with an commutative property for their diagrams between their derived categories. Finally we establish some examples that are equivalents under different QFT approach due to that all its elements have the same hypercohomology too.

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