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SINGULARITIES, TORSION, CAUCHY INTEGRALS AND THEIR SPECTRA ON SPACE-TIME

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ABSTRACT. All field sources are identified as fields ϕ_{AB}, which can be identified too as poles or singularities in the complex Riemannian manifold model of the space-time including field sources, such that their integrals can calculate their value through the Cauchy type integrals as the Conway integrals to any loop generated in the local causal structure of the space-time around of these fields. The integrals are solutions of the spinor equation associated to the corresponding twistor field equation. A theorem is mentioned on the evidence of field torsion as field invariant and geometrical invariant in poles of Cauchy type integrals in spinor-twistor frame. Then an immediate result is that torsion existence in the space-time induces gravitational waves in a projective bundle. Sources are evidence at least locally of torsion existence. Then exists curvature here. Some conjectures and technical lemmas are mentioned as references of other works and is included a new application conjecture too.

1. Introduction

Remember that the Universe is a unfinished source of energy, because the Universe is itself the universal source of energy. In the interstellar process of the formation of the sidereal objects are observed different manifestations of the interactions of particles that give shape to the themselves sidereal objects and the existence to all field that is present in the space-time giving arising to its causality and holonomy of this in the future steps. Then if we consider the sources of these fields from a microscopic level using their spinor frame, we can characterize this sources as monopoles or multi-poles which could be used as sources in the space-time where the torsion can be obtained by the values of its integrals. However, beyond of these considerations we need establish the existence of a field observable related to the space agitation and the values of the field sources related with this space agitation (waves).

Remember that a field observable that is studied in the electromagnetic fields with the interaction of matter of the space-time is the torsion. Likewise, in last studies of the torsion, have been established many aspects as strong relations with the CPT-violating, quantum gravity production and the twistor frame used in the study of the ruled surfaces to minimal surfaces in string theory and their moduli stacks. Also there is indicium from the fermions. In the string theory is evident that each undulation (or string) can be viewed as a twistor in a duality defined in an integrable system [1].

After, studies related with strictly geometry of the physical stacks suggest that the sources are the poles that can conform a field carpet of the space-time always that the correspondence between twistor space and space-time be bi-univocal and determined through an integral operators cohomology between both spaces [2]. This is precisely the idea of

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the twistor geometry of Penrose [3]. Also this open to more research in integral geometry. The important aspect where to start is determine and define the connection and its corresponding bundle with a determined form whose measure is defined on which we can realize the integration. For example, in the aspect of singularities, in the managing of monopoles, is defined a spin connection. These monopoles have a magnetic nature and its spin bundle is the space [4]:

$$\Theta^{A'} = \{\theta^{AA'} \in SL(2, \mathbb{C}), d(\theta^{AA'}) = (1/2)T^{AA'}, \tag{1}$$

However, do us the following question: how is possible to calculate around of sources or holes (in each case) the cohomological functionals of such luck that the integrals $\int \omega$ are contour integrals whose value is determined by the residue theorem and determines evidence of field torsion?

How to evidence the curvature of a Universe modeled as a complex Riemannian manifold with singularities interpreted these as sources or sinks through the value of its integrals?

Can be generated loops from a causal structure of the space-time around of the fields "sources" that evidence said curvature?

The answer is yes to everything, if we consider the fields "sources or holes" inside the corresponding light cones having locally the field characterized as poles. Then loops are closed curves around of pole that represent co-cycles of spinor-waves arising from an electromagnetic field that with the gravity can be identified in a Cauchy type integral, possible considering these poles as fields sources. The values collection of these integrals (of said sources) are added to other integrals on space-time geodesics and determine the curvature of the space-time.

However, to talk of co-cycles we have that to identify a dual space where the spectrum of curvature can be identified as perturbations in the space-time from the energy states in a Hamiltonian manifold whose spectra are Higgs bosons [5] or any another particle type that represents a deformation in microscopic context of the space-time [6] as singularity. Then the singularities can be measurable (that are poles in Cauchy integrals) as the value of cohomological functionals given by the residue value when certain class of Cauchy integrals are evaluated in these points. But, in twistor framework these poles as singularities of the complex Riemannian manifold, which are represented as the dual elements called spinors, which can reveal torsion and curvature. Then twistor-spinor sources implies space-time undulations, which implies torsion and its contour integrals curvature values.

2. TORSION ENERGY

The following result obtained in [6, 7], is the fundamental principle that is required to gauge and detect the torsion through of the tensor $A_{\alpha\beta}$, [1] considering the law transformation to pass from a field Z^{α}, to other Z^{β}, to two coordinate system α, and β, to transform the surface Σ, through the transformation law (taking the diagram (12)):

$$\Sigma_{\alpha\beta} = A_{\alpha\beta} I^{\beta\gamma} \Sigma_{\gamma\alpha}, \tag{2}$$

We have the following theorem

Theorem 1. *(F. Bulnes, Y. Stropovsvky, I. Rabinovich). We consider the embedding*

$$\sigma : \Sigma \to (\mathbb{T}(S) \otimes \mathbb{T}(S))^{*}, \tag{3}$$

[1] Kinematic twistor tensor.

The space $\sigma(\Sigma)$, is smoothly embedded in the twistor space $(\mathbb{T}(S) \otimes \mathbb{T}(S))^*$. Then their curvature energy is the energy given in the interval $M_N \geq A_{\alpha\beta} Z^{\alpha} I^{\beta\gamma}$
$Z_{\gamma} \geq 0$.

6. □

The before criteria established by the theorem, is an integral geometry criteria considering the values of the integrals of the contours around of singularities (holes or sources) which in the twistor-spinor framework will be of the form [7-10]:

$$\Psi = \int_{S^1} \frac{\partial}{\partial \omega^A} \frac{\partial}{\partial \omega^B} \frac{\partial}{\partial^C} \frac{\partial}{\partial \omega^D} \in_{AB} \in_{BC} \pi d\pi + \int_{S^1} \pi_A \pi_B \pi_C \pi_D \in_{A'B'} \in_{B'C'} d\pi, \qquad (4)$$

which is deduced of the following natural conjecture, in $\mathbb{M}$, a complex Riemannian manifold:

Conjecture (F. Bulnes) 2. 1. The curvature in spinor-twistor framework can be perceived with the appearing of the torsion and the anti-self-dual fields.

6. □

3. INTEGRALS OF CAUCHY TYPE

We consider the screw effect due the torsion and determine a $2D$-numerical model whose re-interpretation in the cosmology objects can be the black holes or sources as stars, even galaxies. For example, the behavior intersidereal magnetic alignment of galaxies, using 2-dimensional complex surfaces considering the Morera′s and Cauchy-Goursat's theorems [8] to be evaluated and can apply in an numerical program. Likewise, for example, on singularities or poles in the space-time, considering the space-time a complex Riemannian manifold with singularities. This could be represented by the surface of the real part of a function $h(z) = \frac{z+3}{(z-1)(z-2)}$. The moduli of these points are at least 2 and thus lie inside one contour defined inside the circle $C :\mid z \mid= 3$. Likewise, the contour integral can be split into two smaller integrals using the Cauchy-Goursat theorem having finally the contour integral [8].

$$\int_C h(z)dz = \int_C \left(\frac{z+3}{(z-1)(z-2)} \right) dz = 2\pi i(-4) + 2\pi i(5) \in \mathbb{C}, \qquad (5)$$

(see the figure 1)). Likewise, the value is a traditional cohomological functional element of this element is a contour around the singularity as can be viewed in the figure 1).

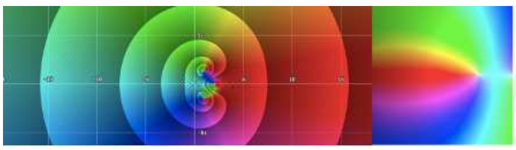


FIGURE 1. Poles or singularities of the complex function $h(z) = \frac{z+3}{(z-2)(z-1)}$. The surface folds around of their singularities or poles. The two pictures refer to the same singularities or poles.

The two pictures refer to the same singularities or poles.

We consider the real space-time $\mathbb{M}$, as real component of complexified Minkowski space $\mathbb{CM}$, represented as (complex projective) lines in $\mathbb{PT}$, which will be all points of the complexified compactified Minkowski space $\mathbb{CM}^{\#}$. Then these lines are spheres in projective

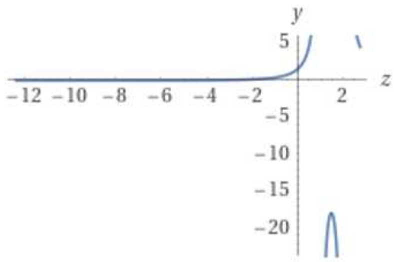


FIGURE 2. Graph representation of the function $h(z)$.

space, since those lines that lie in $\mathbb{PN}$, represent points of the real space-time $\mathbb{M}$, (possibly at infinity), but since these lines still complex projective lines, they are indeed Riemann spheres [3, 11] (see the figure 3).

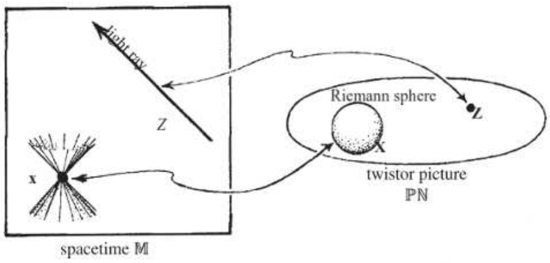


FIGURE 3. Twistor correspondence: a ray Z in Minkowski space $\mathbb{M}$ corresponds to a point in $\mathbb{PN}$; a point x of $\mathbb{M}$ corresponds to a Riemann sphere S, in $\mathbb{PN}$.

We consider the spin group of complex Minkowski space given by $\mathfrak{so}(2,4)$, (rotations in two directions) whose Lie algebra $\mathfrak{so}(2,4)$, is isomorphic to $SU(2,2)$. Likewise, electromagnetic waves in conformal actions of the group $SU(2,2)$ on a dimensional flat model of the space-time, can be described to two auto-dual Maxwell fields of positive and negative frequency [11]. In this case we can use structures of light cones and the spinor framework to obtain that conformal theories of gauge fields as electromagnetic fields measure other fields, for example gravity.

We call twistor function $f(Z^\alpha) = \frac{1}{A_\alpha Z^\alpha}$, $\forall Z^\alpha \in \mathbb{T}$. Here A_α, defines a line or ray directed by the field Z^α, in the twistor space.

Lemma 1. *Actions of* $SU(2,2)$*, act on* $\mathbb{M}$*, to generate twistor functions.*

Proof. There is a demonstration realized in [12]. □

However, a demonstration more direct is through twistor transform, where are generated in a natural way the twistor functions from actions of $SU(2,2)$. In the demonstration realized in [12], was considered the Penrose anti-transform of

$$\phi_{A'B'\ldots C'}(x) = \frac{1}{2\pi i}\int_\Gamma \pi_{A'}\ldots\pi_{C'}[\rho_x f(Z^\alpha)]\pi_{E'}d\pi^{E'},$$

where ρ_x, denotes restriction to the celestial sphere of space-time point $x^{AA'}$, in $\mathbb{PT}$. Then in the space-time the structure of light cones happens (as is established in the figure

4) where the two cones (future and past light cones) can be characterized, considering a pair of homegeneity -4 twistor functions $\{f_{-4}^{(1,2)}\}$ which will necessarily correspond to electromagnetic solutions $\phi_{A'B'}^{(1,2)}$, to complete the solution in $SU(2,2)$ of the space-time by the integral (4), for example. Likewise, $SU(2,2)$, acts on a 2-fold cover $\mathbb{M}^{\#4}$, of $\mathbb{M}^{\#2}$. Likewise, $\mathbb{M}^{\#4}$, is therefore a 4-fold cover to $\mathbb{M}^{\#}$.

Combinations of them with a homogeneity -2 function (corresponding to a space-time scalar ϕ) to make a homogeneity -6 function can be generated to give solutions in gravity.

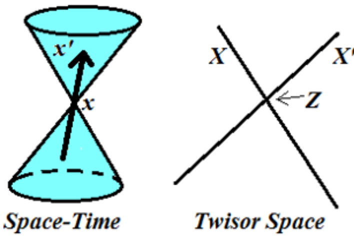


FIGURE 4. $\mathbb{M} \cong \mathbb{CP}^3$.

The before lemma established an important space-time relationship between electromagnetic, scalar and gravitational fields. This relation of space-time can be described through torsion considering the poles as field sources. Then here can be incorporated the established: the sources of the field are the poles in $s_{rev}(x)$, with multiplicity $-\lambda$, and $s_{adv}(x)$, with multiplicity $1-\lambda$ and described these poles by a cohomology of contours. Then we have integrals of Cauchy type to their evaluation.

The values corresponding to dipoles can be obtained by these integrals, which are of Cauchy type.

Likewise using the material of the past section, we recall the Conway integral as those integrals of the form [12]:

$$\phi(x) = \frac{1}{\pi i} \oint \frac{\rho(s)ds}{(x - y(s))^2}, \tag{6}$$

which yields the solutions $\phi_\lambda(x)$, described as the "contours" (cohomological functionals with integrals $\int \omega$) used surrounds the pole with the corresponding multiplicities λ. That to the fields represented by (6) in the Higgs fields[2].

Indeed, we have spin solutions in the spin bundle $\Theta^{A'}$, which are the magnetic monopoles. These solutions are achieved in the space defined in (1).

Then we obtain a twistor function which represents the field $\phi_\lambda(x)$, in the following way: Consider the function ξ^A, of the parameter s, and the twistor $(\omega^A, \pi_{A'})$, defined as:

$$\xi^A(s) = \omega^A - iy^{AA'}(s)\pi_{A'}, \tag{7}$$

Then ω^A, can form a twistor function $f(\omega^A, \pi_{A'})$, by performing the contour integral

$$f(\omega^A, \pi_{A'}) = \frac{1}{\pi i} \oint \frac{\alpha_A \beta^A \rho(s)ds}{\alpha_B \xi^B(s) \beta_C \xi^C(s)}, \tag{8}$$

where α_A, β_A, are arbitrating fixed spinors. f, is homogeneous of degree -2, and one recovers the Conway integral form by use of the s.

[2]Of fact the denominator of its rational function complies in $\mathbb{R}^3$, the formulation $\lambda(1- \mid \phi \mid^2)^2$, in the Higgs fields

One expects any massless field on M^I/γ, to be the same of a source-free field and a field which can be expanded in terms of the multi-pole fields just described in the cohomological space $H^1_{\mathcal{L}}(U'', \mathcal{O}(-2))$, (special case when $h = 0$) when $\mathcal{L}$, is a part of the ruled surface inside the world line in the causal structure of the space-time.

Theorem 2. *(F. Bulnes) 3. 1. The twistor function (8) involves torsion through the poles of its contour integral.*

Proof. See demonstration details in [12] □

In the demonstration was used the torsion result in [9], which establishes formally that:

$$\tau(t_1, t_2) = \pm 2\pi \mathrm{cot}\Psi \mathrm{F}\{K(t_1, t_2)\}\delta(t_1, t_2), \tag{9}$$

that is to say, in a pole spectra there is a delta function [9, 15] that acts as screw effect producing undulations in the space-time. This evidences the torsion. We need transform to the spinor-twistor context the functional relation, having as a momentum-space function[3] $f(\pi_\alpha, \pi_{\alpha'})$.

There is a twistor transform which can produce in a natural way twistor functions as was mentioned in the lemma 3. 1.

We considered the curvature energy as such function then we have:

$$k(\omega^\alpha, \pi_{\alpha'}) = \int_{-\infty}^{\infty} K(\pi_\alpha, \pi_{\alpha'}) e^{-\omega^0\pi_0 - \omega^1\pi_1} d\pi_0 d\pi_1, \tag{10}$$

where have been considered coordinates systems in the momentum-space given by $(\pi_\alpha, \pi_{\alpha'})$, and its spectra will be a twistor space whose coordinate system is $(\omega^\alpha, \pi_{\alpha'})$, where this last, involves spinors ω^α, which define the undulations.

Then details of torsion in the demonstration carry us to

$$\begin{aligned}\tau(\omega^\alpha, \pi_{\alpha'}) &= \frac{c}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 2\pi \mathrm{F}\{K(\pi_\alpha, \pi_{\alpha'})\}\delta(\omega^\alpha, \pi_{\alpha'}) d\pi_0 d\pi_1 \\ &= \frac{2\pi c}{2\pi} 2\pi k(\omega^\alpha, \pi_{\alpha'})\delta(\omega^\alpha, \pi_{\alpha'}) \\ &= \pm 2\pi \mathrm{cot}\Psi k(\omega^\alpha, \pi_{\alpha'})\delta(\omega^\alpha, \pi_{\alpha'}),\end{aligned} \tag{11}$$

which evidences the existence of torsion in a twistor function. Then the torsion involves spinors which define the undulations in the space-time. Which is its source? Its source is the field $\phi_{A'B'\ldots C'}$, which by the lemma 3. 1, was established as a (linearized) gravity solution in space-time, implying an important space-time relationship between electromagnetic, scalar and gravitational fields. These are gravitational waves.

The supermassive object is the singularity of the space-time $\mathbb{M}$, which deforms $\mathbb{M}$, undulating it.

[3]Theorem. The Fourier transform of a momentum-space function $f(\pi_\alpha, \pi_{\alpha'})$, is the integral transform:

$$\begin{aligned} f(\omega^\alpha, \pi_{\alpha'}) &= \frac{1}{(-2\pi i)}\int_{-i\Gamma_\sigma} f(\pi_\alpha, \pi_{\alpha'}) e^{-\omega^0\pi_0 - \omega^1\pi_1} d\pi_0 d\pi_1 \\ &= \sum_{a,b=0}^{\infty} C_{ab}(\pi_\alpha, \pi_{\alpha'}) \frac{\mathrm{sgn}\sigma\Gamma(a+1)\Gamma(b+1)}{(-2\pi i)^2(\omega^0)^{a+1}(\omega^1)^{b+1}}, \end{aligned}$$

$$\forall(\omega^0, \omega^1) \in \mathbb{C}^* \times \mathbb{C}^*.$$

Conjecture (F. Bulnes) 3. 1. Any source or hole that displaces and rotates around its axis in the space-time generates torsion.

12. □

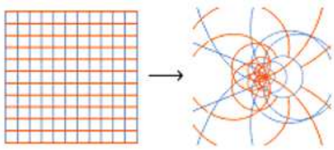
Corollary (F. Bulnes) 3. 1. The torsion existence in the space-time induces gravitational waves in a complex projective bundle of the space-time.

From a point of view of the physical facts, a projective structure can choice the class of preferred paths in space-time. For another side, we know that the space-time is affected by a supermassive body like a singularity in the space-time (can be a black hole or a particle like source), deforming it and creating a high superposition energy from said object [13, 14]. But also the torsion as has been showed in [12, 13], produce undulations in the space-time similar to the produced by singularities (black holes) in the space-time. However these undulation travel in the infinity of the space-time beyond of Newtonian limit of the gravitational events horizon, being then an space with hyperbolic structure where a specific kind of non-Euclidean metric lies in projective space. One can measure the projective structure by use of massive test bodies. This shows that the space-time is undulated under presence of a supermassive body interpreted as source or singularity in terms of Riemannian geometry.

The equation $\nabla^{\alpha} T^{\alpha\beta} = 0$, where $T^{\alpha\beta}$, is the energy-mass tensor, is the permanent energy-matter production.

By the Conjecture (F. Bulnes) 3. 1 [12], which says that any source or hole that displaces and rotates around its axis in the space-time generates torsion. The corollary results inmediate.

Proof. We consider the twistor function (of wave-function) $f(\omega^A, \pi_{A'})$, where by the theorem 3. 1, involves torsion in the poles of this function. The spinors in $\mathbb{PT}$, can be perceived as the circles in the mapping. □



that is to say,

$$\mathbb{C} \to \mathbb{PT}, \tag{12}$$

where the superprojective $\mathbb{PT}$, has the circles $\mathbb{P}^1(\mathbb{C})$, which give a sphere.

Likewise, the space obtained by the embedding:

$$S^2\Sigma, \tag{13}$$

produces in $\mathbb{PT}$, twistors that are in the surface $(\mathbb{T}(S) \otimes \mathbb{T}(S))^*$, with corresponding spinors [6, 12] $\omega_A\omega_B$, which defines undulations from microscopic level.

Then the torsion involves spinors which define the undulations in the space-time. Finally by the lemma 3.1, was established as a (linearized) gravity solution in space-time, implying an important space-time relationship between electromagnetic, scalar fields and gravitational fields. These are gravitational waves. □

From astrophysics studies to evidence the torsion [14] from sources existence we can consider the following integral of indices of the fields (particles/anti-particles):

$$n_X - n_X = \frac{g}{(2\pi)^3} \int d^3p \left(\frac{1}{1 + e^{E_X/T}} - \frac{1}{1 + e^{E_X}/T} \right), \tag{14}$$

which suggest that particle/anti-particle asymmetry can thus be generated from torsion.

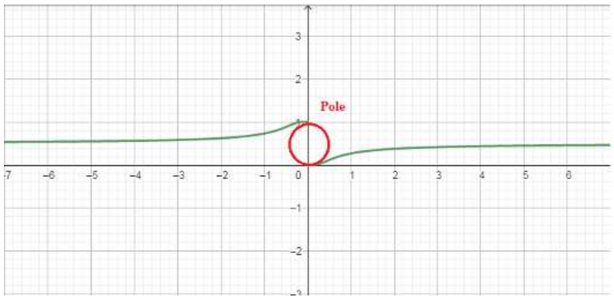


FIGURE 5. Function $f(x) = \frac{1}{(2\pi)^3} \frac{1}{1+e^{\frac{E_X}{T}}}$, to the particle/anti-particle

asymmetry model of a singularity generated by fermions [14].

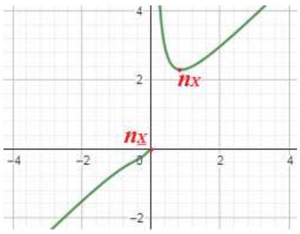


FIGURE 6. Integral of indices given by (16).

4. APPLICATIONS

We consider experiments in mathematical-physics consigned in an electronic device consisting of a Hall sensor, a magnetic dilaton (at way of particle in movement) and vertical rotation and horizontal movements device emulating the conic trajectory of dilaton (see the experiment [13]).

Lemma 2. *The spectral torsion (torsion energy) on the conic trajectory of dilaton obtained for the Hall sensor is:*

$$\tau(\omega_1, \omega_2) = \frac{1}{2\gamma} \frac{1}{(a^2 + x^2)} \frac{b}{r} TLSa^2(\omega_1, \omega_2), \tag{15}$$

15. □

The torsion formula includes the function $Sa(\omega) = \frac{\sin\omega}{\omega}$, which evidences the torsion existing in the space where happens the dilaton movement and the interaction with the electromagnetic field of the Hall sensor. The rotation energy is given by the implicit torsion in the rotations of the sensor device.

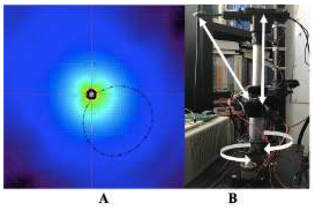


FIGURE 7. Magnetic sensing. A). The magnetic field lobules in green-clear blue represent the magnetic sensing along of two cycles of the conic spiral trajectory of the dilation in upside (also see the Fig. 3.4B). Device of Hall sensor chip using the three synchronize movements to permits an inverted cone as well as right cone [15].

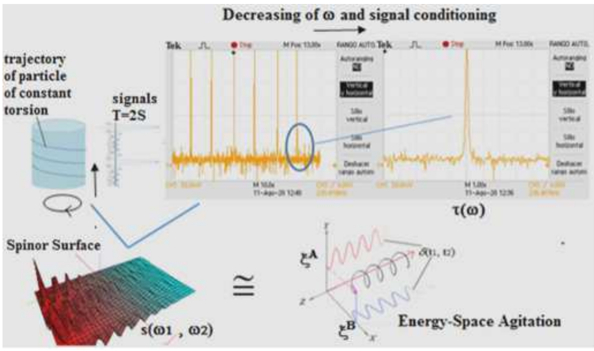


FIGURE 8. Complete method by Hall Effect sensor for detecting of field torsion [15].

Also in this study of poles and singularities of space-time obeying the structure of the complex manifold $\mathbb{M}$, with singularities, we have a recent discover by James Webb telescope, where the singularities are sources of energy, and the torsion is evident also.

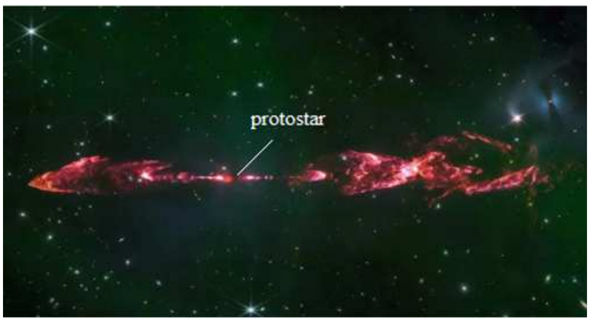


FIGURE 9. JWST-Telescope's image of the protostar, which is hidden. Towards the center of this object, called HH212, a star is being born that it is probably no more than 50,000 years old [16]. The picture was taken in Hydrogen-light.

We can actually see the brightness of the proto-star because it is hidden within a dense rotating disk gas and dust (see the figure 9). This confirms the existence of torsion by the conjecture 3. 1. Also, can be seen the reddish pink jets which shoots in opposite polar directions, such as is predicted in litter values given by the integral (16) and showed by the figure 5.

Also, confirms the prolongation on space due the circles $\mathbb{P}^1(\mathbb{C})$, in the sphere possibly the proto-star which produce diverse particle products, fermionic currents, and gravitational waves in the space $(\mathbb{T}(S) \otimes \mathbb{T}(S))^*$.

5. CONCLUSIONS

If we consider the multi-poles as the sources of the fields of different nature of the space-time (of fact their moduli stack is obtained by equivalences in field theory using some *gerbes* of derived categories), we can to use the loops around of these poles as contours of the cohomological functionals. Likewise, these cohomological functionals establish classes of contour integrals that determine and evaluate through the residue theorem and other as the Morera′s and Cauchy-Goursat′s theorems where from a point of view of the physics are the value of the states of a field. These values can be registered to design and determine spinor waves which, considering the covariant nature of these invariant objects and the local structure of the space-time, carry us to integrability conditions from the curvature of the space-time and therefore its torsion as second curvature.

In this work has been related three fundamental aspects of the space-time, this modeled as a complex Riemannian manifold including singularities as poles of corresponding twistor functions, and complex momentum-space functions; the poles, spinors and torsion, which has been demonstrated along the paper that these aspects are related in duality, equivalence and realization (of its corresponding representation through twistor framework). These relations were proved in the theorem 3. 1. Likewise, any spin pair s, and $-s$, can generate rotations in a local region $\mathcal{Q}$, of the space-time $\mathcal{M}$, considering the spinors as undulations from electromagnetic field acting on the space. In this paper has been proved that a sufficient condition to the torsion existence is the existence of spinors. In another work [18, 19], and using the construction of monopoles from twistor space, has been demonstrated fundamentally that

$$\{\pm 1\} \to \mathrm{Spin}(\mathcal{Q}) \to SO(\mathcal{Q}), \tag{16}$$

where $\mathrm{Spin}(\mathcal{Q}) \to GL(\mathcal{Q})$, which could demonstrate the necessity, that is to say, to start with spinors to obtain rotations which are effects of torsion. Evidences found in the Universe were showed in the applications section [16, 17] (see the figure 9).

The Conway integrals can be considered in axisymmetric boundaries and also non-axisymmetric cases where gravity aspects are confined within axisymmetric boundaries, for example locally. The potential and attractions in the classical gravitational fields for the elementary thin disc can be given in closed form in terms of elliptic integrals and elementary functions. However, if we consider the electromagnetic nature of the iso-rotations (in $SO(2)$, for example) for fields as sources, we need other formulism based in twistor geometry, where elliptic integrals are analogues in the space $H^1_{\mathcal{L}}(U'', (-2))$, that is to say, to purely electromagnetic fields. Then the cohomological analogous are "poles" which can be interpreted as '"sources" of field (see the figures 9 and 10). Then is had the conjecture 3.1. Finally, and considering the microscopic deformations of the space-time, we can conclude that the torsion is always present through a spin connection. Important representations of momentum-space functions are obtained through spectra in twistor space, which evidences the torsion as element of the twistor function.

The fields obtained by the contour integrals around of sources represent the fields induced by another field (in this case could be the gravity) in an extended region of the space-time. This is foreseen in theoretical physics and fine electronics experiments to field torsion.

Here the integrals of the class (3) are due to the fields of electromagnetic type (Actions of $SU(2,2)$. act on $\mathbb{M}$). If we want involve the gravitational sources we need an integral of the form:

$\phi_{A'B'}(x) = \frac{1}{2\pi i}\oint\limits_{\Gamma} \pi_{A'}\pi_{B'}F_h(Z)\pi_{B'}d\pi^{B'}$

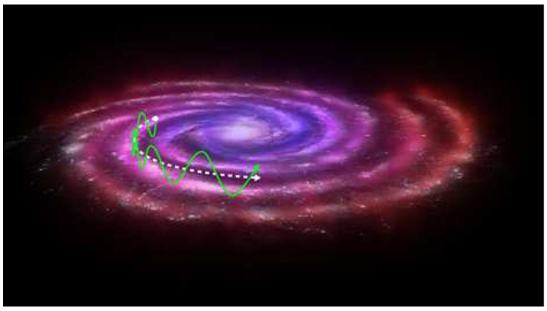


FIGURE 10. The cohomological analogous are "poles" which can be interpreted as "sources" of electromagnetic radiative energy. The spinor formalism could be used through the responsible electromagnetic energy of the accretion and iso-rotation of a sidereal object as a galaxy, interacting with matter, for example.

SOME NOTATION

M^I - Real affine Monkowski space when I, is the line defined by $\phi_{\pm}$, on $\mathbb{M}^{\pm}$, in $\phi_\lambda(x)$.

$H^1_{\mathcal{L}}(U'', \mathcal{O}(-2))$ - Cohomological space of first integrals of Conway type.

$\mathbb{PT}$ - Projective twistor space. Said space is isomorphic as a complex manifold to $\mathbb{P}^3(\mathbb{C})$. In this case the complex manifold to $\mathbb{P}^3(\mathbb{C})$, is $\mathbb{CM}^{\#}$.

ϕ - Solutions or integrals of the equation $\nabla^\alpha T^{\alpha\beta} = 0$, in the form $\Box\phi = 4\pi\rho(s)$.

$\mathbb{CM}^{\#}$ - Complexified compactified Minkowski space.

ω^A, ω_A - Covariant and contra-variant spinors.

$Z_\alpha = (\omega^A, \pi_{A'})$ - Twistor field.

$Z_\alpha = (\omega_A - ix_{AA'}\pi^{A'})$ - Associate spinor field to twistor field, called Robinson field whose flagpole is the Robinson congruence associated with the twistor.

π^A, π_A - Covariant and contra-variant twistors (or twistors-spinors).

$\phi_{A_1{}'\ldots A'_{2h}}(x)$ - Spin field of spinors $A_1{}', \ldots, A^1_n$, with helicit h.

$f(Z)$ -Homogeneous twistor function.

$\omega^{\alpha\alpha'}$ - four vectors components of spinorial momentum.

$\phi_{A'B'}(x) = \frac{1}{2\pi i}\oint\limits_{\Gamma} \pi_{A'}\pi_{B'}F_h(Z)\pi_{B'}d\pi^{B'}$ - 4-dimensional case we have the Penrose transform to celestial sphere $\mathbb{CP}^1$.

$\mathbb{P}^1(\mathbb{C})$ - Complex projective space of one dimension. The complex projective space $\mathbb{P}^1(\mathbb{C})$, is the Riemann sphere.

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