TJMM
14 (2022), No. 1, 35-42

ON THE PRACTICAL STABILIZATION OF INFINITE-DIMENSIONAL PERTURBED SYSTEMS

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ABSTRACT. In this paper, we investigate the notion of practical feedback stabilization of a class of non-autonomous infinite-dimensional systems. Assuming appropriate conditions on the perturbation term, it is shown that if every frozen-time control system is stabilizable then the corresponding non-autonomous infinite-dimensional control system is practically stabilizable. Sufficient conditions for the practical feedback stabilizability on a separable Hilbert space are given. This approach is based on the freezing method. Some examples are considered to illustrate the result obtained.

1. INTRODUCTION

In the literature on control theory of time-varying dynamical systems, stabilization it is one of the important properties of the system and has attracted many researchers, see [3, 8, 10, 11, 12, 14, 15, 16]. Lyapunov function approach and the method based on spectral decomposition are the most widely used techniques for studying stabilizability of special classes of control systems, see [3, 5, 9, 17]. However, the freezing method has become well known among these techniques. In particular, it has been used to prove the exponential stabilizability of nonlinear control systems in Banach space, see [11]. Sufficient conditions for the practical stabilization of infinite-dimensional evolution equations in Banach spaces have been developed [5]. In the infinite-dimensional control systems, the investigation of practical stabilization is more complicated and require more sophisticated techniques. The practical stabilization is to find the state feedback candidate such that the solution of the closed-loop system is practically exponentially stable in the Lyapunov sense in which the origin is not necessary an equilibrium point. In this case, the authors proved the practical feedback stabilization of the time-varying control systems in Hilbert spaces where the nominal system is a linear time-varying control systems globally null-controllable and the perturbation term satisfies some conditions, see [3]. Moreover, the stabilizability conditions are obtained by solving a Ricatti differential equation and do not involve any stability property of the evolution operator. In the other hand, for time-varying control systems in finite-dimensional spaces, we showed that the system is practical stabilizable if the linear time-varying control system is uniformly controllable and the nonlinear perturbation satisfies some conditions, see [4].
In this paper, we will apply the freezing technique to investigate the practical feedback stabilization of a class of non-autonomous infinite-dimensional control systems with estimation for the semi-group generated by the leading coefficient of the system in Banach spaces. Moreover, sufficient conditions of practical stabilizability on a separable Hilbert spaces are established. The result of the paper can be considered as further extensions of Medina [11] when the origin is not necessary an equilibrium point. A practical stability

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2010 *Mathematics Subject Classification.* 34H15, 37N35, 47D03, 47D06.
Key words and phrases. Banach spaces, Infinite-dimensional control systems, Freezing method, Linear operator, Practical stabilization.

approach is obtained. Finally, we provide some special cases and examples to illustrate the results obtained.
The remainder of this paper is organized as follows. In Section 2 some preliminary results are summarized. Sufficient stabilizability conditions in a Banach space are established in Section 3. Explicit conditions for stabilizability on a separable Hilbert space are given in Section 4. Our conclusion is given in Section 5.

2. Mathematical Preliminaries

We will use the following notation throughout this paper: $\mathbb{R}_+$ denotes the set of all non-negative real numbers, X denotes a Banach space with the norm $\|.\|$. I is the identity operator. A^* denote the the adjoint of the operator A. Also, we denote by

- $L(X)$ (respectively, $L(X,Y)$) is the Banach space of all linear bounded operators T mapping X into X (respectively, X into Y) endowed with the norm
$$\|T\| = \sup_{x\in X} \frac{\|T(x)\|}{\|x\|}\cdot$$
- $L_\infty(\mathbb{R}_+, L(X,Y))$ is the space of all mappings $S : \mathbb{R}_+ \longrightarrow L(X,Y)$, with the norm
$$\|S\|_\infty = \sup_{t\in\mathbb{R}} \|S(t)\|.$$

Let X and U be real or complex infinite-dimensional Banach spaces. We consider the control dynamical system
$$\begin{cases} \dot{x} = A(t)x + B(t)u + \omega(t,x), \quad t \geq 0, \\ \\ x(0) = x_0, \end{cases} \tag{1}$$
where $x \in X$ is the system state, $u \in U$ is the control input, $A \in L_\infty(\mathbb{R}_+, L(X))$ and $B \in L_\infty(\mathbb{R}_+, L(U,X))$. Next, we are interested in suitable feedback of the form
$$u(t) = L(t)x(t), \tag{2}$$
where $L \in L_\infty(\mathbb{R}_+, L(X,U))$. Then, the equation (1) takes form
$$\dot{x} = W_L(t)x + \omega(t,x), \quad t \geq 0 \tag{3}$$
with the initial condition $x(0) = x_0 \in X$ and $W_L = A + BL \in L_\infty(\mathbb{R}_+, L(X))$ have a dense constant domain $D(W_L(t)) = D_L \subseteq X,\ t \geq 0$.
We can rewrite equation (3) in the form
$$\dot{x} = W_L(\tau)x + [W_L(t) - W_L(\tau)]x + \omega(t,x), \tag{4}$$
for an arbitrarily fixed $\tau \geq 0$.
With the non-autonomous equation (4) we associate the integral equation
$$x(t) = e^{tW_L(\tau)}x_0 + \int_0^t e^{(t-s)W_L(\tau)}[(W_L(s) - W_L(\tau))x(s) + \omega(s,x(s))]ds, \quad 0 \leq t \leq T$$
with a fixed $\tau \geq 0$.
Now, we define the notion of practical stabilizability when the origin is not necessarily an equilibrium point of the system (1).

Definition 1. *The system (1) is called practically stabilizable if there exists a continuous feedback control $u : X \to U$, such that system (1) with $u(t) = u(x(t))$ satisfies*
$$\|x(t)\| \leq k\|x_0\|e^{-\lambda t} + r, \quad \forall t \geq 0,$$
where $\lambda > 0$, $k \geq 0$ and $r > 0$.

3. Practical stabilization

The purpose of this section is to establish the practical stabilization of (1) in Banach spaces. We start by studying the practical stabilization of nonlinear control time-varying systems based on the freezing method. We shall suppose the following assumptions:

($\mathcal{A}_1$): The nonlinear operator $\omega : \mathbb{R}_+ \times X \to X$ is continuous, there exists a non-negative constant ω_0, such that $\|\omega(t,0)\| \leq \omega_0$, for all $t \in \mathbb{R}_+$ and satisfying the following inequality

$$\|\omega(t,x) - \omega(t,y)\| \leq \gamma \|x-y\|, \quad \forall t \geq 0, \quad x, y \in X, \quad \gamma > 0, \tag{5}$$

($\mathcal{A}_2$): The operator W_L satisfies the Lipschitz property

$$\|W_L(t) - W_L(s)\| \leq \widetilde{q}|t-s|, \quad t, s \geq 0$$

where $\widetilde{q}$ is a positive constant independent of t, s.
The strongly continuous semi-group $e^{W_L(\tau)t}$ satisfies

($\mathcal{A}_3$): $\Theta(W_L(.), \omega) = \displaystyle\int_0^\infty (t\widetilde{q} + \gamma) \sup_{\tau \geq 0} \|e^{W_L(\tau)t}\| dt < 1.$

($\mathcal{A}_4$): $\mu = \displaystyle\int_0^\infty \sup_{\tau \geq 0} \|e^{W_L(\tau)t}\| dt < \infty$ and $\xi = \displaystyle\sup_{t \geq 0} \sup_{\tau \geq 0} \|e^{W_L(\tau)t}\| < \infty.$

We assume that for any $x_0 \in X$, the corresponding integral equation of (1) exists and is unique, see [13].
Let's go now to present the practical stabilizability theorem in the practical case, it consists on finding a feedback control $u(t) = L(t)x(t)$ for keeping the closed-loop system (3) practically uniformly exponentially stable.

Theorem 1. *Suppose that there exists an operator $L \in L_\infty(\mathbb{R}_+, L(X, U))$ satisfies conditions $(\mathcal{A}_1) - (\mathcal{A}_4)$, then the system (1) is practically stabilizable by means of the feedback law (2). Moreover, any solution $x(t)$ of equation (3) satisfies the inequality*

$$\|x\|_0 \leq \frac{\xi \|x_0\| + \omega_0 \mu}{1 - \Theta(W_L(.), \omega)}. \tag{6}$$

Proof. Suppose that $x(t)$ is the solution of (4). Then, we obtain for all $t \in [0, \infty)$

$$\|x(t)\| \leq \xi \|x_0\| + \int_0^t \Gamma(t-s)(\widetilde{q}|\tau - s| + \gamma)\|x(s)\| ds + \omega_0 \int_0^t \Gamma(t-s) ds$$

where

$$\Gamma(t) = \sup_{\tau \geq 0} \|e^{tW_L(\tau)}\|.$$

Taking $t = \tau$, we have

$$\|x(\tau)\| \leq \xi \|x_0\| + \int_0^\tau \Gamma(\tau - s)(\widetilde{q}(\tau - s) + \gamma)\|x(s)\| ds + \omega_0 \int_0^\tau \Gamma(\tau - s) ds.$$

Then, for any finite t_0,

$$\sup_{0 \leq \tau \leq t_0} \|x(\tau)\| \leq \xi \|x_0\| + \sup_{0 \leq \tau \leq t_0} \|x(\tau)\| \int_0^{t_0} \Gamma(t_0 - s)(\widetilde{q}(t_0 - s) + \gamma) ds + \omega_0 \int_0^{t_0} \Gamma(t_0 - s) ds.$$

But,

$$\int_0^{t_0} \Gamma(t_0 - s)(\widetilde{q}(t_0 - s) + \gamma) ds \leq \int_0^\infty \Gamma(t_1)(\widetilde{q} t_1 + \gamma) dt_1 = \Theta(W_L(.), \omega)$$

and

$$\int_0^{t_0} \Gamma(t_0 - s) ds \leq \int_0^\infty \Gamma(t_1) dt_1 = \mu.$$

Therefore,

$$\sup_{t\leq t_0} \|x(t)\| \leq (\xi\|x_0\| + \omega_0\mu)(1-\Theta(W_L(.),\omega))^{-1}.$$

Since, t_0 is arbitrary, this inequality provide the practical Lyapunov stability. To establish the practical exponential stability, we define the new variable

$$x_\varepsilon(t) = e^{\varepsilon t}[x(t) - rI], \tag{7}$$

where r is a positive constant, ε is a positive real parameter small enough and $x(t)$ is a solution of equation (3). Substituting (7) into (3), we get

$$\dot{x}_\varepsilon(t) = (\varepsilon I + W_L(t))x_\varepsilon(t) + \omega_1(t, x_\varepsilon(t)) \tag{8}$$

where

$$\omega_1(t,h) = e^{\varepsilon t}\omega(t, he^{-\varepsilon t} + rI) + W_L(t)re^{\varepsilon t}, \quad h \in X.$$

The assumption $(\mathcal{A}_1)$ yields

$$\|\omega_1(t,h) - \omega_1(t,k)\| \leq \gamma\|h-k\|, \quad \forall h,k \in X, \quad \forall t \geq 0.$$

Applying our reasoning above to equation (8), we obtain according to (6) that $x_\varepsilon(t)$ is a bounded function. Therefore, the relation (7) implies the practical uniform exponential stability of the system (3). This ends the proof of Theorem 1. □

Remark 1. *The phrase pair ε is a positive real parameter small enough means that ε have to be a real parameter such that the assumption $(\mathcal{A}_3)$ be satisfied.*

4. Systems in Hilbert spaces

The previous Theorem in earlier section show that the extension of the freezing method to evolution equations is based on norm estimates for relevant semi-groups. Moreover, obtaining these estimates is difficult. So, we will restrain ourselves by equations in separable Hilbert spaces.

To express the next results, let H be a separable Hilbert space and A a linear compact operator acting in H. If $\{e_k\}_{k=1}^{\infty}$ is an orthogonal basis in H and the series $\sum_{k=1}^{\infty} < Ae_k, e_k >$ converges, then the sum of the series is called the trace of the operator A and is denoted by

$$Trace(A) = Tr(A) = \sum_{k=1}^{\infty} < Ae_k, e_k > .$$

Definition 2. *An operator A satisfying the relation $Tr(A^\star A) < \infty$ is said to be a Hilbert-Schmidt operator, where A^* is the adjoint operator of A.*

The norm

$$N_2(A) = N(A) = \sqrt{Tr(A^\star A)}$$

is called the Hilbert-Schmidt norm of A.

Theorem 2. *(See [2]) A linear compact operator T acting on a separable Hilbert space H is a Hilbert-Schmidt operator if, and only if, there is an orthogonal normal basis $\{e_k\}_{k=1}^{\infty}$ in H, such that*

$$\sum_{k=1}^{\infty} \|Te_k\|^2 < \infty.$$

In this case, the quantity $\sum_{k=1}^{\infty} \|Te_k\|^2 < \infty$ is independent of the choice of the orthogonal normal basis $\{e_k\}_{k=1}^{\infty}$.

Theorem 3. *(See [7]) Let A be a Hilbert-Schmidt operator, then the inequality*

$$\|e^{At}\| \leq e^{\alpha(A)t} \sum_{k=0}^{\infty} \frac{t^k g^k(A)}{(k!)^{\frac{3}{2}}}, \quad \forall t \geq 0$$

holds, where $\alpha(A) = \sup Rel\sigma(A)$, $g(A) = [N^2(A) - \sum_{k=1}^{\infty} |\lambda_k(A)|^2]^{\frac{1}{2}}$, $\lambda_k(A)$ are the eigenvalues including their multiplicities and $\sigma(A)$ is the spectrum of the operator A.

The theorem below is a corollary of theorem 1 in separable Hilbert spaces.

Theorem 4. *Suppose that there exists an operator $L \in L_\infty(\mathbb{R}_+, L(H,U))$ satisfies conditions $(\mathcal{A}_1)$ and $(\mathcal{A}_2)$ with $W_L \in L_\infty(\mathbb{R}_+, L(H))$ is a Hilbert-Schmidt operator and*

$$\sum_{k=0}^{\infty} \frac{v_L^k}{\sqrt{k!}} \left[\frac{(k+1)\widetilde{q}}{\rho_L^{k+2}} + \frac{\gamma}{\rho_L^{k+1}} \right] < 1, \tag{9}$$

where

$$\rho_L = -\sup_{\tau \geq 0} \alpha(W_L(\tau)) > 0, \quad v_L = \sup_{\tau \geq 0} g(W_L(\tau)) < \infty.$$

Then, the system (1) in closed-loop with the linear feedback (2) is practically uniformly exponentially stable.

Proof. By Theorem 3, we have

$$\|e^{W_L(\tau)t}\| \leq \wedge(t, W_L(.)), \quad t, \tau \geq 0,$$

where

$$\wedge(t, W_L(.)) = e^{-\rho_L t} \sum_{k=0}^{\infty} \frac{v_L^k t^k}{(k!)^{\frac{3}{2}}}, \quad t \geq 0.$$

By assumptions $(\mathcal{A}_1)$ and $(\mathcal{A}_2)$, we have

$$\|x(t)\| \leq \wedge(t, W_L(.))\|x_0\| + \int_0^t \wedge(t-s, W_L(.))(|\tau - s|\widetilde{q} + \gamma)\|x(s)\|ds + \omega_0 \int_0^t \wedge(t-s, W_L(.))ds.$$

Taking $t = \tau$, we have

$$\|x(\tau)\| \leq \Gamma\|x_0\| + \int_0^\tau \wedge(\tau - s, W_L(.))[(\tau - s)\widetilde{q} + \gamma]\|x(s)\|ds + \omega_0 \int_0^\tau \wedge(\tau - s, W_L(.))ds,$$

where

$$\Gamma = \sup_{\tau \geq 0} \wedge(\tau, W_L(.)).$$

Then, for any finite T,

$$\sup_{0 \leq \tau \leq T} \|x(t)\| \leq \Gamma\|x_0\| + \sup_{0 \leq \tau \leq T} \|x(t)\| \int_0^T \wedge(T-s, W_L(.))[(T-s)\widetilde{q} + \gamma]ds + \omega_0 \int_0^T \wedge(T-s, W_L(.))ds,$$

We have,

$$\begin{aligned} \int_0^t \wedge(t-s, W_L(.))(t-s)ds &= \int_0^t e^{-\rho_L(t-s)} \sum_{k=0}^{\infty} \frac{v_L^k (t-s)^{k+1}}{(k!)^{\frac{3}{2}}} ds \\ &\leq \int_0^\infty e^{-\rho_L z} \sum_{k=0}^{\infty} \frac{v_L^k z^{k+1}}{(k!)^{\frac{3}{2}}} dz = R_0(W_L), \quad t \geq 0, \end{aligned}$$

where

$$R_0(W_L) = \sum_{k=0}^{\infty} \frac{(k+1)v_L^k}{\sqrt{k!}\rho_L^{k+2}} < \infty.$$

In addition,

$$\begin{aligned}\int_0^t \wedge(t-s, W_L(.))ds &= \int_0^t e^{-\rho_L(t-s)} \sum_{k=0}^{\infty} \frac{v_L^k (t-s)^k}{(k!)^{\frac{3}{2}}} ds \\ &\le \int_0^{\infty} e^{-\rho_L z} \sum_{k=0}^{\infty} \frac{v_L^k z^k}{(k!)^{\frac{3}{2}}} dz = R_1(W_L), \quad t \ge 0.\end{aligned}$$

where

$$R_1(W_L) = \sum_{k=0}^{\infty} \frac{v_L^k}{\sqrt{k!}\rho_L^{k+1}} < \infty.$$

Then,

$$\sup_{t\le T} \|x(t)\| \le \Gamma\|x_0\| + \sup_{t\le T} \|x(t)\|[\widetilde{q}R_0(W_L) + \gamma R_1(W_L)] + \omega_0 R_1(W_L).$$

Thus, from the condition (9), we obtain

$$\sup_{t\le T} \|x(t)\| \le (1 - \widetilde{q}R_0(W_L) - \gamma R_1(W_L))^{-1}(\Gamma\|x_0\| + \omega_0 R_1(W_L)).$$

Since T is arbitrary, this relation implies the Lyapunov stability. To prove the practical exponential stability of system (3) on a separable Hilbert space, it is sufficient to proceed in the same way as in the proof of Theorem 4, we get the result. This ends the proof. □

In the rest of this section, we prove the practical stabilization of (1) where the nominal system is time-invariant in separable Hilbert spaces.
We consider the following system

$$\dot{x} = Ax + Bu(t) + \omega(t,x), \quad t \ge 0, \tag{10}$$

where $x \in H$ $u \in U$, A and B are constant operator, $A \in L(H)$, $B \in L(U,H)$ and $\omega : \mathbb{R}_+ \times H \to H$ is a nonlinear function. Thus, Theorem 4 implies this result.

Lemma 1. *Suppose that there exists an operator $L \in L(H,U)$, such that $\rho_L = -\sup \alpha(W_L) > 0$ and $v_L = g(W_L)$ where $W_L = A + BL$ is a Hilbert-Schmidt operator. Under assumption $(\mathcal{A}_1)$, if*

$$\gamma \sum_{k=0}^{\infty} \frac{v_L^k}{\sqrt{k!}\rho_L^{k+1}} < 1,$$

then the system (10) in closed-loop with the linear feedback $u(t) = Lx(t)$ is practically uniformly exponentially stable.

Let Δ the set of all the pair $(t,s) \in \mathbb{R}_+^2$ with $t \ge s$.

Definition 3. *(Evolution operator)*
A mapping $U : \Delta \to L(X)$ is called evolution operator on X, if

- (i) $U(t,t) = I, \quad \forall t \ge 0,$
- (ii) $U(t,\sigma)U(\sigma,s) = U(t,s), \quad \forall t \ge \sigma \ge s \ge 0,$
- (iii) *$U(t,s)x$ is jointly continuous with respect to t,s for every $x \in X$.*

We state the following well-known controllability criterion for infinite-dimensional control system.

Definition 4. *([1],[8]) The system $[A(t),B(t)]$ is globally null-controllable in finite time if and only if*

$$\exists T > 0, c > 0 : \int_{t_0}^{T} \|B^*(s)U^*(T,s)x^*\|^2 ds \ge c\|U^*(T,t_0)x^*\|^2, \quad \forall x^* \in X^*,$$

where $U(t,s)_{t\geq s\geq 0}$ be an evolution operator of the generator $A(t)$.

Next, we give an example to illustrate the applicability of our result.

Example 1. *We consider the following system in separable Hilbert space*

$$\dot{x} = \sin(\sigma t)Ax(t) + Bu(t) + \omega(t,x), \quad t \geq 0, \quad \sigma > 0 \tag{11}$$

where A and B are a compact operators and $\omega : \mathbb{R}_+ \times H \to H$ is a continuous operator in (t,x) and verifies the condition (5). So, $\tilde{A}(t) = \sin(\sigma t)A$ and $B(t) = B$. We choose A, such that $(\tilde{A}(t), B)$ is globally null-controllable in finite time $t \geq 0$ and the operator $L(t)$, such that

$$\rho_L = -\sup_{\tau\geq 0} \alpha(W_L(\tau)) > 0, \text{ and } v_L = \sup_{\tau\geq 0} g(W_L(\tau)) < \infty,$$

with $W_L = \tilde{A} + BL$ is a Hilbert-Schmidt operator. Moreover, we have $\widetilde{q} = |\sigma|\|A\|$. Due to the hypothesis $(\mathcal{A}_3)$, we obtain the following result.

Lemma 2. *Under assumption $(\mathcal{A}_1)$ if the inequality*

$$\sum_{k=0}^{\infty} \frac{v_L^k}{\sqrt{k!}} \left[\frac{(k+1)|\sigma|\|A\|}{\rho_L^{k+2}} + \frac{\gamma}{\rho_L^{k+1}} \right] < 1.$$

holds, the system (11) in closed-loop with the linear feedback $u(t) = L(t)x(t)$ is practically uniformly exponentially stable.

Example 2. *Consider the second order system*

$$\begin{cases} \dot{x_1} = (-1 - e^{-t})x_1 + (2 + e^{-t})x_2 + e^{-t}u(t) + \frac{1}{2\pi}\arctan(t)x_1 + \frac{e^{-t}}{\sqrt{1+t^2}} \\ \dot{x_2} = \frac{-1-2t}{1+t}x_1 - \frac{1}{1+t}x_2 - \frac{1}{1+t}u(t) + \frac{1}{2\pi}\arctan(t)x_2, \quad t \geq 0 \end{cases} \tag{12}$$

where $x^T = (x_1, x_2)$, $u \in \mathbb{R}$ is control input.
Rewrite (12 in the form (1) with
$A(t) = \begin{pmatrix} -1 - e^{-t} & 2 + e^{-t} \\ \frac{-1-2t}{1+t} & -\frac{1}{1+t} \end{pmatrix}$, $B(t) = \begin{pmatrix} e^{-t} \\ -\frac{1}{1+t} \end{pmatrix}$ *and*

$$F(t,(x_1,x_2)) = \frac{\arctan(t)}{2\pi} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{e^{-t}}{\sqrt{1+t^2}} \\ 0 \end{pmatrix}.$$

Moreover, the function F satisfies the assumption $(\mathcal{A}_1)$ with $\gamma = \frac{1}{4}$.
Let $L(t) = \begin{pmatrix} 1 & -1 \end{pmatrix}$. Then,

$$W_L(t) = \begin{pmatrix} -1 & 2 \\ -2 & 0 \end{pmatrix}.$$

We conclude that the conditions of Theorem 4 are hold with $\rho_L = v_L = 1$ and $\widetilde{q} = 0$. Therefore, the system (12) in closed-loop with the linear feedback $u(t) = L(t)x(t)$ is practically uniformly exponentially stable.

5. Conclusion

Practical stabilization of a class of non-autonomous infinite-dimensional systems has been investigated. These results become explicit to control systems on a separable Hilbert space. Illustrative examples are given to indicate significant improvements and the application of the results.

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