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ON SOME COMMON FIXED POINTS THEOREMS IN INTUITIONISTIC FUZZY METRIC SPACES AND ITS APPLICATIONS

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Abstract. In this paper, we established some common fixed point theorems for pairs of semi compatible and occasionally weakly compatible mappings in an intuitionistic fuzzy metric space (briefly IFM space) satisfying contractive type condition. In this paper, we observe that the notion of common property $(E.A.)$ relatively relaxes the required containment of the range of one mapping into the range of other which is utilized to construct the sequence of joint iterates. We extended the results established in [13] to intuitionistic fuzzy metric space.

1. Introduction

It is no doubt true that the concept of fuzzy set given by Zadeh [37] helped other researchers to introduce and develop the fuzzy metric in distinct forms. Credit for further development goes to Grabiec [10] who extended two fixed point theorems of Banach [6] and Edelstein [9] for contractive mappings of complete and compact fuzzy metric space in the sense of Kramosil and Michalek [19]. Thereafter, George and Veeramani ([11, 12]) modified the concept of fuzzy metric space given by Kramosil and Michalek [19] and defined a Hausdroff topology on it.
Attansov ([3, 4]) generalized the fuzzy metric space to intuitionistic fuzzy metric space and proved some common fixed point theorems for the same. Park [24] in 2004, using the idea of intuitionistic fuzzy sets defines the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and t-conorm as a generalization of fuzzy metric space due to George and Veermani ([11, 12]). Various authors proved common fixed point theorems in different spaces by taking under consideration the mappings satisfying contractive type of conditions. Majority of the results comprises of either commuting or weak commutativity mappings introduced by Sessa [27] while establishing common fixed point results. In 1994, Mishra et al. [23] extended the notion of compatible maps introduced by Jungck et al. [16] in metric space under the name of asymptotically commuting maps to fuzzy metric spaces. Sharma et.al.([29, 30]) introduced and studied the concept of common fixed point for weakly and multivalued mappings in intuitionistic fuzzy metric spaces. In 2002, Aamir and Moutawakil [5] established a couple of new common fixed point theorems under strict contractive conditions via (E.A.) property. Pant and Pant [25] explored the new horizons by establishing the common fixed points theorems for a pair of non compatible maps and the property E.A in fuzzy metric space. For more details, we refer to (Alaca et al. [1], Anderson et al. [2], Deng [7], Erceg[8], Grabiec [10], Jungck [14], Kaleva and Seikkala [18], Kubiaczyk and Sharma [20], Jungck ([14, 15]), Jungck and Rhoades [17], Schweizer and Skaler[26], Sessa [27], Sharma and Bamboria [28], Sharma and Deshpande [31], Thagafi

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and Shahzad [33], Singh and Jain [34], Turkoglu et. al. [35]), Manro et.al. [22], Pant [25] and Sharma et. al. [29]) respectively.

2. Preliminaries

Definition 1. *[26] A binary operation* $*: [0,1] \times [0,1] \to [0,1]$ *is continuous t-norm if* $*$ *is satisfying the following conditions:*

$$* \, is \, commutative \, and \, associative, \tag{1}$$

$$* \, is \, continuous, \tag{2}$$

$$a * 1 = a, \forall \, a \in [0,1], \tag{3}$$

$$a * b \leq c * d \, whenever \, a \leq c \, and \, b \leq d, \forall \, a, \, b, \, c, \, d \, \in [0,1]. \tag{4}$$

Definition 2. *[26] A binary operation* $\Diamond : [0,1] \times [0,1] \to [0,1]$ *is continuous t-norm if* $\Diamond$ *is satisfying the following conditions:*

$$\Diamond \, is \, commutative \, and \, associative, \tag{5}$$

$$\Diamond \, is \, continuous, \tag{6}$$

$$a \Diamond 0 = a, \forall \, a \in [0,1], \tag{7}$$

$$a \Diamond b \leq c \Diamond d \, whenever \, a \leq c \, and \, b \leq d, \forall \, a, \, b, \, c, \, d \in [0,1]. \tag{8}$$

Remark 1. *The concept of triangular norms (t norms) and triangular conorms (t-conorms) are known as the axiomatic sekeltons that we use for characterizing fuzzy intersections and unions respectively. These concepts were originally introduced by Menger* [21] *in his study of Statistical metric spaces.*

Definition 3. *[1] A 5-tuple* $(X, M, N, *, \Diamond)$ *is said to be an intuitionistic fuzzy metric spaces if* X *is an arbitrary set,* $*$ *is a continuous t-norm,* $\Diamond$ *is a continuous t-conorm and* M, N *are fuzzy sets on* $X^2 \times [0, \infty)$ *satisfying the following conditions:*
$\forall x, y, z \in X$ *and* $t, s > 0$,

$$M(x,y,t) + N(x,y,t) \leq 1, \tag{9}$$

$$M(x,y,0) = 0, \tag{10}$$

$$M(x,y,t) = 1, \forall t > 0 \; iff \; x = y, \tag{11}$$

$$M(x,y,t) = M(y,x,t), \tag{12}$$

$$M(x,y,t) * M(y,z,s) \leq M(x,z,t+s), \tag{13}$$

$$M(x,y,.) : [0,\infty) \to [0,1] \; is \; left \; continuous, \tag{14}$$

$$lim_{t \to \infty} M(x,y,t) = 1, \tag{15}$$

$$N(x,y,0) = 1, \tag{16}$$

$$N(x,y,t) = 0, \forall t > 0 \; iff \; x = y, \tag{17}$$

$$N(x,y,t) = N(y,x,t), \tag{18}$$

$$N(x,y,t) \Diamond N(y,z,s) \geq N(x,z,t+s), \tag{19}$$

$$N(x,y,.) : [0,\infty) \to [0,1] isrightcontinuous, \tag{20}$$

$$lim_{t \to \infty} N(x,y,t) = 0. \tag{21}$$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2. *Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \Diamond)$ such that t-norm $*$ and t-conorm $\Diamond$ are associated , i.e., $x\Diamond y = 1 - ((1-x) * (1-y)), \forall x, y \in X$.*

Example 1. *[32] Let (X, d) be a metric space. Define t-norm $a * b = min\{a, b\}$ and t-conorm $a\Diamond b = max\{a, b\}$ and $\forall x, y \in X$ and $t > 0, M_d(x, y, t) = \frac{t}{t+d(x,y)}, N_d(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$. Then $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d, the standard intuitionistic fuzzy metric.*

Example 2. *[1] Let $X = N$. Define $a*b = max\{0, a+b-1\}$ and $a\Diamond b = a+b-ab, \forall a, b \in [0, 1]$ and let M and N be the fuzzy sets on $X^2 \times (0, \infty)$ as follows :*

$\forall x, y \in X and t > 0$. Then $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzy metric space.

$$M(x, y, t) = \begin{cases} \frac{x}{y}, & \text{if } x \le y \\ \frac{y}{x}, & \text{if} y \le x \end{cases}$$

and

$$N(x, y, t) = \begin{cases} \frac{y-x}{y}, & \text{if } x \le y \\ \frac{x-y}{x}, & \text{if} y \le x \end{cases}$$

Remark 3. *In an intuitionistic fuzzy metric space $X, M(x, y, .)$ is non- decreasing and $N(x, y, .)$ is non-increasing, $\forall x, y \in X$.*

Definition 4. *[1] Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then*

A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $lim_{n\to\infty}\{x_n\} = x)$ $if, \forall t > 0$,
$lim_{n\to\infty} M(x_n, x, t) = 1,\ lim_{n\to\infty} N(x_n, x, t) = 0$.

A sequence $\{x_n\}$ in X is said to be a cauchy sequence if $\forall\, t > 0$ and $p > 0$
$lim_{n\to\infty} M(x_{n+p}, x, t) = 1,\ lim_{n\to\infty} N(x_{n+p}, x, t) = 0$.

Definition 5. *[1] An intutionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every cauchy sequence in X is convergent. It is called compact if every sequence contains a convergent subsequence.*

Remark 4. *[1] Since $*$ and $\Diamond$ are continuous, the limit is uniquely determined from (13) and (19), respectively.*

Definition 6. *[34] A pair (A, S) of self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be semi-compatible if $lim_{n\to\infty} M(ASx_n, Sz, t) = 1$, and $lim_{n\to\infty} N(ASx_n, Sz, t) = 0$, for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z$ for some $z \in X$.*

Definition 7. *[32]A pair (A, S) of self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to satisfy $(E.A)$ property if there exists a sequence $\{x_n\}$ in X suchthat $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z$ for some $z \in X$.*

Definition 8. *Two pairs (A, S) and (B, T) of self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are said to share the common property $(E.A)$ if there exists two sequences $\{x_n\}$ and $\{y_n\}$in X suchthat $lim_{n\to\infty} Bx_n = lim_{n\to\infty} Tx_n = lim_{n\to\infty} Ay_n = lim_{n\to\infty} Sy_n = z$ for some $z \in X$.*

Definition 9. *[1]Two self-maps A and S on set X are said to be weakly compatible if they commute at their coincidence point.*

Definition 10. *[33] Two self maps f and g of a set X are called occasionally weakly compatible iff there is a point $x \in X$ which is coincidence point of f and g at which f and g commute.*

Definition 11. *[36] Let Ψ be the class of all non decreasing mappings $\psi : [0,1] \times [0,1] \to [0,1]$ and $\eta : [0,1] \times [0,1] \to [0,1]$ such that*

$$lim_{\to\infty}\psi^n(s) = 1, \forall s \in (0,1] \tag{22}$$
$$\psi(s) > s, \forall s \in (0,1) \tag{23}$$
$$\psi(1) = 1, \tag{24}$$
$$lim_{\to\infty}\eta^n(r) = 0, \forall r \in [0,1) \tag{25}$$
$$\eta(r) < r, \forall r \in (0,1) \tag{26}$$
$$\eta(0) = 0. \tag{27}$$

For examples, we refer to [36].

3. Main results

Theorem 1. *Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Let A, B, S and T be mappings from X into itself such that*

$$M(Ax, By, t) \geq \psi[min\{M(Sx, Ty, t), M(Sx, Ax, t), M(Sx, By, t), M(Ty, Ax, t)\}] \tag{28}$$

and

$$N(Ax, By, t) \leq \eta[max\{N(Sx, Ty, t), N(Sx, Ax, t), N(Sx, By, t), N(Ty, Ax, t)\}] \tag{29}$$

for all $x, y \in X$, where $\psi, \eta \in \Psi$. Also, suppose the pair (A, S) and (B, T) share the common property $(E.A)$, and $S(X)$ and $T(X)$ are closed subsets of X, then the pair (A, S) as well as (B, T) have a coincidence point. Further, A, B, S and T have a unique common fixed point provided the pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible.

Proof. Since the pair (A, S) and (B, T) satisfies the property $(E.A)$. Then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}By_n = lim_{n\to\infty}Ty_n = z$, for some $z \in X$. Since $S(X)$ is closed subset of X , therefore $lim_{n\to\infty}Sx_n = z \in S(X)$ and there is a point u in X such that $Su = z$.
Now, we claim that $Au = z$. If not, then by using (28) *and* (29), we have

$$M(Au, By_n, t) \geq \psi[min\{M(Su, Ty_n, t), M(Su, Au, t), M(Su, By_n, t), M(Ty_n, Au, t)\}],$$

$$N(Au, By_n, t) \leq \eta[max\{N(Su, Ty_n, t), N(Su, Au, t), N(Su, By_n, t), N(Ty_n, Au, t)\}]$$

Letting $n \to \infty$, we have

$$M(Au, z, t) \geq \psi[min\{M(z, z, t), M(z, Au, t), M(z, z, t), M(z, Au, t)\}] = \psi[M(z, Au, t)] > M(z, Au, t),$$

$$N(Au, z, t) \leq \eta[max\{N(z, z, t), N(z, Au, t), N(z, z, t), N(z, Au, t)\}] = \eta[(z, Au, t)] < N(z, Au, t),$$

which are contradictions. Hence $Au = z$. Thus, we have $Au = Su$ i.e. u is coincidence point of the pair (A, S).
Since $T(X)$ is a closed subset of X, therefore $lim_{n\to\infty} Ty_n = z \in T(X)$ and there is a point $w \in X$ such that $Tw = z$. Again using (28) and (29), we obtain

$$M(Ax_n, Bw, t) \geq \psi[min\{M(Sx_n, Tw, t), M(Sx_n, Ax_n, t), M(Sx_n, Bw, t), M(Tw, Ax_n, t)\}],$$

$$N(Ax_n, Bw, t) \leq \eta[max\{N(Sx_n, Tw, t), N(Sx_n, Ax_n, t), N(Sx_n, Bw, t), N(Tw, Ax_n, t)\}]$$

Letting $n \to \infty$, we have

$M(z, Bw, t) \geq \psi[min\{M(z, z, t), M(z, z, t), M(z, Bw, t), M(z, z, t)\}] = \psi[M(z, Bw, t)] > M(z, Bw, t)$,

$N(z, Bw, t) \leq \eta[max\{N(z, z, t), N(z, z, t), N(z, Bw, t), N(z, z, t)\}] = \eta[(z, Bw, t)] < N(z, Bw, t)$,

which are contradictions. Hence $Bw = z$. Thus, we have $Bw = Tw$ i.e. w is coincidence point of the pair (B, T).
Since, the pair (A, S) is semicompatible , so $lim_{n\to\infty} ASx_n = Sz$ and $lim_{n\to\infty} ASx_n = Az$. Also, the limit in an intuitionistic fuzzy metric space is unique, therefore $Az = Sz$.
Next, we'll show that z is a common fixed point of the pair (A, S).
By using (28) *and* (29), we have

$$M(Az, Bw, t) \geq \psi[min\{M(Sz, Tw, t), M(Sz, Az, t), M(Sz, Bw, t), M(Tw, Az, t)\}],$$

$$N(Az, Bw, t) \leq \eta[max\{N(Sz, Tw, t), N(Sz, Az, t), N(Sz, Bw, t), N(Tw, Az, t)\}]$$

Letting $n \to \infty$, we have

$M(Az, z, t) \geq \psi[min\{M(Az, z, t), M(Az, Az, t), M(Az, z, t), M(z, Az, t)\}] = \psi[M(Az, z, t)] > M(Az, z, t)$,

$N(Az, z, t) \leq \eta[max\{N(Az, z, t), N(Az, Az, t), N(Az, z, t), N(z, Az, t)\}] = \eta[(Az, z, t)] < N(Az, z, t)$.
This gives $Az = z$. Thus $Az = z = Sz$. Since w is a coincidence point of B and T and the pair (B, T) is occasionally weakly compatible, so we have $BTw = TBw$ implies $Bz = Tz = z$.

For uniqueness, Let v be anothe fixed point of A, B, S and T. Set $x = z$ and $y = v$ in (28) and (29), we have

$$M(Az, Bv, t) \geq \psi[min\{M(Sz, Tv, t), M(Sz, Az, t), M(Sz, Bv, t), M(Tv, Az, t)\}],$$

$$N(Az, Bv, t) \leq \eta[max\{N(Sz, Tv, t), N(Sz, Az, t), N(Sz, Bv, t), N(Tv, Az, t)\}]$$

Letting $n \to \infty$, we have

$$M(z,v,t) \geq \psi[min\{M(z,v,t), M(z,z,t), M(z,v,t), M(v,z,t)\}] = \psi[M(z,v,t)] > M(z,v,t),$$

$$N(z,v,t) \leq \eta[max\{N(z,v,t), N(z,z,t), N(z,v,t), N(v,z,t)\}] = \eta[N(z,v,t)] < N(z,v,t),$$

this implies $z = v$. Thus z is a unique common fixed point of A, B, S and T. □

Theorem 2. *Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space . Let A, B, S and T be mappings from X into itself such that for all $x, y \in X, a, b, c \geq 0, q > 0$ and $q < a + b + c$,*

$$qM(Ax, By, t) \geq aM(Ty, Sx, t) + bM(Sx, By, t) + cM(Ax, By, t) \tag{30}$$

+max{ M(Ax, Sx,t), M(By,Ty, t)} and

$$qN(Ax, By, t) \leq aN(Ty, Sx, t) + bN(Sx, By, t) + cN(Ax, By, t) \tag{31}$$

+min{ N(Ax, Sx,t), N(By,Ty, t)}
Also, suppose the pair (A, S) and (B, T) share the common property $(E.A)$, and $S(X)$ and $T(X)$ are closed subsets of X, then the pair (A, S) as well as (B, T) have a coincidence point. Further, A, B, S and T have a unique common fixed point provided the pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible.

Proof. Since the pair (A, S) and (B, T) satisfies the property $(E.A)$. Then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} By_n = lim_{n\to\infty} Ty_n = z$, for some $z \in X$. Since $S(X)$ is closed subset of X , therefore $lim_{n\to\infty} Sx_n = z \in S(X)$ and there is a point u in X such that $Su = z$.
Now, we claim that $Au = z$. If not, then by using (30) *and* (31), we have,

$qM(Au, By_n, t) \geq aM(Ty_n, Su, t) + bM(Su, By_n, t) + cM(Au, By_n, t) + max\{M(Au, Su, t), M(By_n, Ty_n, t)\}$

$qN(Au, By_n, t) \leq aN(Ty_n, Su, t) + bN(Su, By_n, t) + cN(Au, By_n, t) + min\{N(Au, Su, t), N(By_n, Ty_n, t)\}$.

Letting $n \to \infty$

$$qM(Au, z, t) \geq aM(z, z, t) + bM(z, z, t) + cM(Au, z, t) + max\{M(Au, z, t), M(z, z, t)\}$$

$$qN(Au, z, t) \leq aN(z, z, t) + bN(z, z, t) + cN(Au, z, t) + min\{N(Au, z, t), N(z, z, t)\},$$

$$(q - c)M(Au, z, t) \geq (a + b)M(z, z, t) + 1 > (a + b)M(z, z, t),$$

$$(q - c)N(Au, z, t) \leq (a + b)N(z, z, t) + 0 \leq 0,$$

which gives
$M(Au, z, t) > \frac{(a+b)}{(q-c)} > 1$ and $N(Au, z, t) \leq 0$, for all $t > 0$, this implies that $Au = z$. Hence $Au = Su$ i.e. u is a coincidence point of (A, S). Since, $T(X)$ is a closed subset of X, there exists a point w in X such that $Tw = z$. Set $x = x_n$, $y = w$ in (30) and (31), we have

$qM(Ax_n, Bw, t) \geq aM(Tw, Sx_n, t) + bM(Sx_n, Bw, t) + cM(Ax_n, Bw, t) + max\{M(Ax_n, Sx_n, t), M(Bw, Tw, t)\}$,

$qN(Ax_n, Bw, t) \leq aN(Tw, Sx_n, t) + bN(Sx_n, Bw, t) + cN(Ax_n, Bw, t) + min\{N(Ax_n, Sx_n, t), N(Bw, Tw, t)\}$.

Letting $n \to \infty$

$qM(z, Bw, t) \geq aM(z, z, t) + bM(z, Bw, t) + cM(z, Bw, t) + max\{M(z, z, t), M(Bw, z, t)\}$

$qN(z, Bw, t) \leq aN(z, z, t) + bN(z, Bw, t) + cN(z, Bw, t) + min\{N(z, z, t), N(Bw, z, t)\}$,

$(q - b - c)M(z, Bw, t) \geq aM(z, z, t) + 1 > aM(z, z, t)$,

$(q - b - c)N(z, Bw, t) \leq aN(z, z, t) + 0 \leq 0$,

which gives
$M(z, Bw, t) > \frac{(a)}{(b-q-c)} > 1$ and $N(z, Bw, t) \leq 0$, for all $t > 0$, this implies that $Bw = z$. Hence $Bw = Tw$ i.e. w is a coincidence point of (B, T). Since, the pair (A, S) is semi-compatible , so $lim_{n\to\infty} ASx_n = Sz$ and $lim_{n\to\infty} ASx_n = Az$. Also, the limit in an intuitionistic fuzzy metric space is unique, therefore $Az = Sz$.
Next, we'll show that z is a common fixed point of the pair (A, S).
By setting $x = z, y = w$ in $(30)\, and\, (31)$, we have

$qM(Az, Bw, t) \geq aM(Tw, Sz, t) + bM(Sz, Bw, t) + cM(Az, Bw, t) + max\{M(Az, Sz, t), M(Bw, Tw, t)\}$,

$qN(Az, Bw, t) \leq aN(Tw, Sz, t) + bN(Sz, Bw, t) + cN(Az, Bw, t) + min\{N(Az, Sz, t), N(Bw, Tw, t)\}$.

Letting $n \to \infty$

$qM(Az, z, t) \geq aM(z, Az, t) + bM(Az, z, t) + cM(Az, z, t) + max\{M(Az, Az, t), M(z, z, t)\}$

$qN(Az, z, t) \leq aN(z, Az, t) + bN(Az, z, t) + cN(Az, z, t) + min\{N(Az, Az, t), N(z, z, t)\}$,

$(q - a - b - c)M(Az, z, t) \geq 1$,
$(q - a - b - c)N(Az, z, t) \leq 0$,
which gives
$M(Au, z, t) > \frac{1}{(q-a-b-c)} > 1$ and $N(Au, z, t) \leq 0$, for all $t > 0$, this implies that $Az = z$. Hence $Az = z = Sz$ i.e. z is a fixed point of (A, S). Since w is a coincidence point of B and T and the pair (B, T) is occasionally weakly compatible, so we have $BTw = TBw$ implies $Bz = Tz = z$. Therefore in all $Az = Sz = Bz = Tz = z$ i.e. z is a common fixed point of A, B, S and T. The uniqueness of a common fixed point is an easy consequence of the inequalities (30) and (31). □

4. Applications

Theorem 3. *Let $(X, M, N, *, \diamondsuit)$ be an intuitionistic fuzzy metric space. Let A, B, S and T be mappings from X into itself such that*

$$\int_0^{M(Ax,By,(t)} \phi(t)dt \geq \int_0^{\psi[m(x,y,t)]} \phi(t)dt, \tag{32}$$

$$\int_0^{N(Ax,By,(t)} \phi(t)dt \leq \int_0^{\eta[n(x,y,t)]} \phi(t)dt, \tag{33}$$

for all $x, y \in X$, where $\psi, \eta \in \Psi$,

$m(x,y,t) = min\{M(Sx,Ty,t), M(Sx,Ax,t), M(Sx,By,t), M(Ty,Ax,t)\}$,

$n(x,y,t) = max\{N(Sx,Ty,t), N(Sx,Ax,t), N(Sx,By,t), N(Ty,Ax,t)\}$, *and $\phi : R_+ \to R_+$ is a Lebesgue integrable and summable function such that for each $\epsilon > 0$, $\int_0^\epsilon \phi(t)dt > 0$. Also, suppose the pair (A,S) and (B,T) share the common property $(E.A)$, and $S(X)$ and $T(X)$ are closed subsets of X, then the pair (A,S) as well as (B,T) have a coincidence point. Further, A, B, S and T have a unique common fixed point provided the pair (A,S) is semi-compatible and (B,T) is occasionally weakly compatible.*

Proof. Since the pair (A,S) and (B,T) satisfies the property $(E.A)$. Then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} By_n = lim_{n\to\infty} Ty_n = z$, for some $z \in X$. Since $S(X)$ is closed subset of X, therefore $lim_{n\to\infty} Sx_n = z \in S(X)$ and there is a point u in X such that $Su = z$. Now, we claim that $Au = z$. If not, then by using (32) *and* (33), we have,

$\int_0^{M(Au,By_n,(t)} \phi(t)dt \geq \int_0^{\psi[m(u,y_n,t)]} \phi(t)dt$,

$\int_0^{N(Au,By_n,(t)} \phi(t)dt \leq \int_0^{\eta[n(u,y_n,t)]} \phi(t)dt$,
where
$\psi[m(x,y,t)] = \psi[min\{M(Su,Ty_n,t), M(Su,Au,t), M(Su,By_n,t), M(Ty_n,Au,t)\}]$,

$\eta[n(u,y_n,t)] = \eta[max\{N(Su,Ty_n,t), N(Su,Au,t), N(Su,By_n,t), N(Ty_n,Au,t)\}]$.

So, $\int_0^{M(Au,z,t)} \phi(t)dt \geq \int_0^{\psi[m(u,z,t)]} \phi(t)dt$,

$\int_0^{N(Au,z,(t)} \phi(t)dt \leq \int_0^{\eta[n(u,z,t)]} \phi(t)dt$,

where
$\psi[m(u,z,t)] = \psi[min\{M(z,z,t), M(z,Au,t), M(z,z,t), M(z,Au,t)\}]$
$= \psi[M(z,Au,t)] > M(z,Au,t)$,

$\eta[n(u,z,t)] = \eta[max\{N(z,z,t), N(z,Au,t), N(z,z,t), N(z,Au,t)\}]$
$= \eta[N(z,Au,t)] < N(z,Au,t)$.

i.e., $\int_0^{M(Au,z,t)} \phi(t)dt \geq \int_0^{M(Au,z,t)} \phi(t)dt$,

$\int_0^{N(Au,z,t)} \phi(t)dt \leq \int_0^{N(Au,u,t)} \phi(t)dt$, which are contradictions, these implies $Au = z$. Hence $Au = Su$ i.e. u is a coincidence point of (A,S). Since, $T(X)$ is a closed subset of X, then $\lim_{n\to\infty} Ty_n = z \in T(X)$ and there exists a point w in X such that $Tw = z$. Set $x = x_n$, $y = w$ in (32) and (33), we have
$\int_0^{M(Ax_n,Bw,t)} \phi(t)dt \geq \int_0^{\psi[m(x_n,w,t)]} \phi(t)dt$,

$\int_0^{N(Ax_n,Bw,t)} \phi(t)dt \leq \int_0^{\eta[n(x_n,w,t)]} \phi(t)dt$,

$\int_0^{M(z,Bw,(t)} \phi(t)dt \geq \int_0^{\psi[m(z,w,t)]} \phi(t)dt$,

$\int_0^{N(z,Bw,(t)} \phi(t)dt \leq \int_0^{\eta[n(z,w,t)]} \phi(t)dt,$

where

$\psi[m(z,w,t)] = \psi[min\{M(z,z,t), M(z,z,t), M(z,Bw,t), M(z,z,t)\}]$
$= \psi[M(z,Bw,t)] > M(z,Bw,t),$

$\eta[n(z,w,t)] = \eta[max\{N(z,z,t), N(z,z,t), N(z,Bw,t), N(z,z,t)\}]$
$= \eta[N(z,Bw,t)] < N(z,Bw,t).$

i.e. $\int_0^{M(z,Bw,t)} \phi(t)dt \geq \int_0^{M(z,Bw,t)} \phi(t)dt,$

$\int_0^{N(z,Bw,t)} \phi(t)dt \leq \int_0^{N(z,Bw,t)} \phi(t)dt.$

This gives, $Bw = z$. Hence $Tw = Bw = z$ or w is a coincidence point of the pair (B,T). Since, the pair (A,S) is semicompatible , so $lim_{n\to\infty} ASx_n = Sz$ and $lim_{n\to\infty} ASx_n = Az$. Also, the limit in the intuitionistic fuzzy metric space is unique, therefore $Az = Sz$. Next, we'll show that z is a common fixed point of the pair (A,S).
By setting $x = z, y = w$ in $(32)\, and\, (33)$, we have

$\int_0^{M(Az,Bw,(t)} \phi(t)dt \geq \int_0^{\psi[m(z,w,t)]} \phi(t)dt,$

$\int_0^{N(Az,Bw,(t)} \phi(t)dt \leq \int_0^{\eta[n(z,w,t)]} \phi(t)dt,$

where

$\psi[m(z,w,t)] = \psi[min\{M(Az,z,t), M(Az,Az,t), M(Az,z,t), M(z,Az,t)\}]$

$= \psi[M(Az,z,t)] > M(Az,z,t),$

$\eta[n(z,w,t)] = \eta[max\{N(Az,z,t), N(Az,Az,t), N(Az,z,t), N(z,Az,t)\}]$

$= \eta[N(Az,z,t)] < N(Az,z,t).$

i.e. $\int_0^{M(Az,z,t)} \phi(t)dt \geq \int_0^{M(Az,z,t)} \phi(t)dt,$

$\int_0^{N(Az,z,t)} \phi(t)dt \leq \int_0^{N(Az,z,t)} \phi(t)dt.$

This implies that $Az = z$. Hence $Az = z = Sz$ i.e. z is a fixed point of (A,S). Since w is a coincidence point of B and T and the pair (B,T) is occasionally weakly compatible, so we have $BTw = TBw$ implies $Bz = Tz = z$. Therefore in all $Az = Sz = Bz = Tz = z$ i.e. z is a common fixed point of A, B, S and T. The uniqueness of a common fixed point is an easy consequence of the inequalities (32) and (33).

□

Theorem 4. *Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Let A, B, S and T be mappings from X into itself such that*

$$q \int_0^{M(Ax,By,t)} \phi(t)dt \geq a \int_0^{M(Ty,Sx,t)} \phi(t)dt + b \int_0^{M(Sx,By,t)} \phi(t)dt \qquad (34)$$

$+c\int_0^{M(Ax,By,t)}\phi(t)dt+\int_0^{max\{M(Ax,Sx,t),M(By,Ty,t)\}}\phi(t)dt,$

$$q\int_0^{N(Ax,By,t)}\phi(t)dt\leq a\int_0^{N(Ty,Sx,t)}\phi(t)dt+b\int_0^{N(Sx,By,t)}\phi(t)dt \tag{35}$$

$+c\int_0^{N(Ax,By,t)}\phi(t)dt+\int_0^{min\{N(Ax,Sx,t),N(By,Ty,t)\}}\phi(t)dt,$

for all $x,y\in X$, where $a,b,c\geq 0, q>0, q<a+b+c$ and $\psi: R_+\to R_+$ is a Lebesgue integrable and summable function such that for each $\epsilon>0, \int_0^{\epsilon}\phi(t)dt>0$. Also, suppose the pair (A,S) and (B,T) share the common property $(E.A.)$, and $S(X)$ and $T(X)$ are closed subsets of X, then the pair (A,S) as well as (B,T) have a coincidence point. Further, A,B,S and T have a unique common fixed point provided the pair (A,S) is semi-compatible and (B,T) is occasionally weakly compatible.

Proof. The proof follows easily on the lines of Theorem 3. □

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