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AN UNIQUE AND SYMPLE MATHEMATICAL MODEL FOR THE UNIVERSE EXPANSION AND THE INTRODUCTION OF SOME SWITCHING COEFFICIENTS FOR ITS DEFINITION

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Abstract. In this paper the authors revisit the Mathematical Models of the Universe Expansion previously defined in [1] and [2] and point out some of their shortcomings. In order to remove these shortcomings we elaborate a simple Mathematical Model which is valid for the entire domain of the no dimensional time.

1. Introduction

From the beginning we mention the fact that the variants of the Mathematical Models for the Universe Expansion (UE) previously presented in [1] and [2] used as basic elements the no-dimensional measures δ and τ. With this occasion we remind that δ represents the relation between the Universe dimension at a given moment (denoted with l) and actual maximum dimension (denoted with l_{max}, which has the value $l_{max} = 93 \times 10^9$ light years (l-y). On the other side, τ represents the relation between the time elapsed up to a given moment, from the "explosion" Big-Bang (B-B) denoted with t and the total time elapsed up to present and admitted to be $t_{max} = 13.7 \times 10^9$ years.

Regarding the Mathematical Model presented in [1], it is to mention that it has the disadvantage that it is too complex and assumes very elaborate calculations. On the other side it is valid only for the values in the domain for the no dimensional time $\tau = (0.05\ to\ 1.0)$, thus it can be applied only for a portion of 95% of the entire interval of the time elapsed from B-B up to the present time. The second variant of the Mathematical Model [2] is simpler regarding the calculation complexity, but it has the disadvantage that it can be applied only for a portion of 90% of the entire time interval elapsed from the B-B up to the present, corresponding to the values of the no dimensional time in the domain $\tau = (0.1 to 1.0)$. Considering the facts mentioned above, logically we have the problem to elaborate a new simple Mathematical Model for (UE). This new Mathematical Model must eliminate as much as possible the disadvantages of these two variants mentioned above. The solution of this problem is given in the following sections.

2. A New Simple Mathematical Model for (UE) in the Time Domain $\tau = (0.1\ to\ 1.0)$

In Fig.1, the curve C, given with a thin line, represents the graphical expression of the simple UE mathematical model which is the subject of [2]. This model is valid for the no dimensional time $\tau = (0.1\ to\ 1.0)$ as we mentioned above.

The algebraic expression of the respective mathematical model, established in [2] is the following:

$$\delta_C = 1.96 - \left(1.9044 - \tau^2\right)^{1/2}. \tag{1}$$

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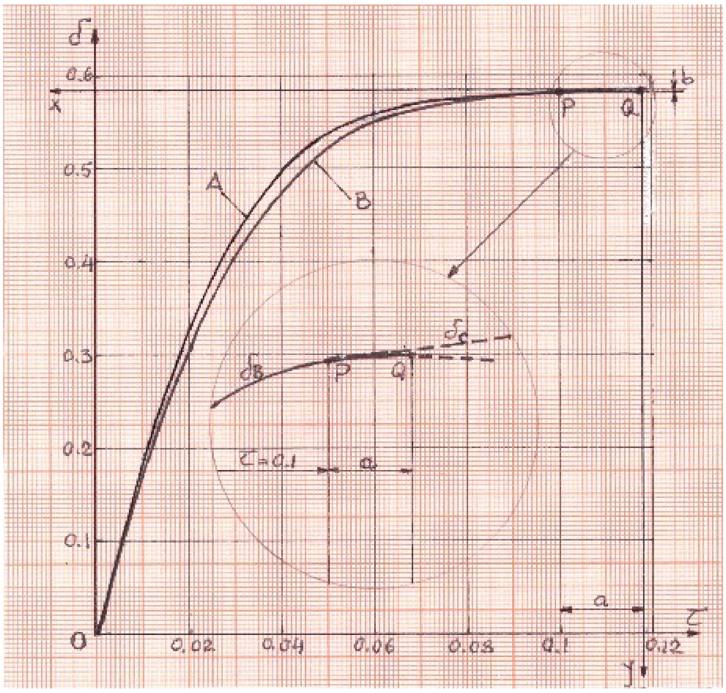

Figure 1.

The curve B, traced with a bold line, represents the graphical expression of the simple mathematical model which we will establish in the next section. This will be valid for the no- dimensional time domain $\tau = (0 \; to \; 0.1)$. The points A are on a curve which imaginarily unite them (for not to affect the quality of the figure). This curve represents the evolution of δ in function of τ, resulted from some measurements performed in Astronomy and cosmological reasoning.

The elements comprised in the medallion of Fig.1 will be presented in what follows.

In continuation, the mathematical reasoning presented is referring to Fig.2.

In order to find easier the characteristics of the curve B of Fig.1, in Fig.2 in tracing this curve we admitted higher values for the representation scales for δ but more especially for τ. Also, in Fig.2, beside the curve B, for comparison, was represented the curve "A" of Fig.1 too, for the domain of the values $\tau = (0 \; to \; 0.1)$. With a view to come near the form of the curve B with a graphical representation of a polynomial function, for the start we adopted the coordinate axis system xQy where Q is the origin of this system. Thus we have, for xQ = 0 and yQ = 0. To establish the corresponding function to B curves we used its likeness with one of the graphical representations studied in [3]. In this way we admitted that a primary version of this function can be:

$$y = \alpha(x)^4. \tag{2}$$

Thus, the point P, having $\tau_P = 0.1$, represents the contact point between the curves B and C (see Fig.1), and for in this point to assure the continuity between these curves it is necessary that two conditions must be satisfied, as we show below. We see that between the points P and Q exists a distance "a" on the abscissa axis direction (Qx) and a distance "b" on the ordinates axis direction (Qy). To make the transition from the function (2) to a form corresponding to curve B represented in coordinate τ and that one with the

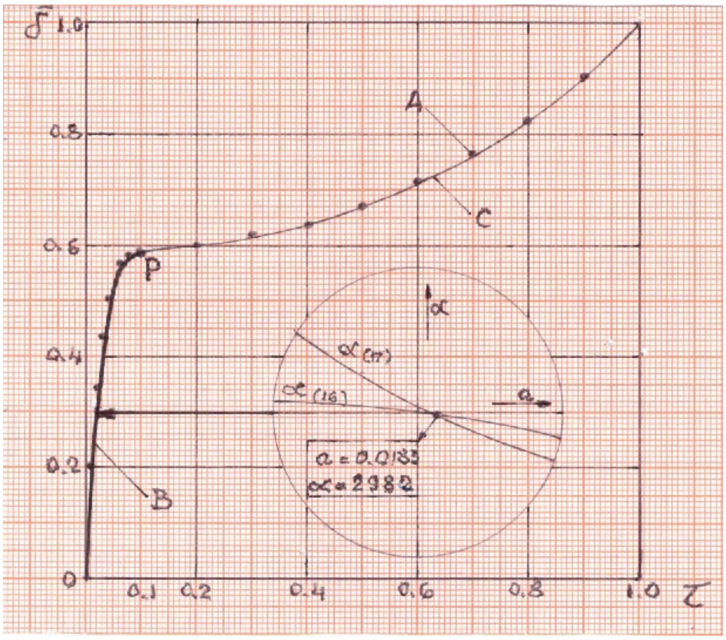

FIGURE 2.

origin in the point Q, we act similarly as in the case of curve C, making the following coordinate change:

$$\tau = 0.1 + a - x \tag{3}$$

$$\delta_B = 0.584 + b - y. \tag{4}$$

If from the equations (3) and (4), we express x as a function of τ and respectively y as a function of δ_B we obtain the following equation which express δ_B as a function of τ and the constants a and b:

$$\delta_B = 0.584 - \alpha(0.1 + a - \tau)^4 + b. \tag{5}$$

In order to find the values of these three "unknowns", right away from the equation (5), that is the value of α, a and b, we must have three equations resulted from the application of the equation (5) in the points with the known characteristics belonging to the curve B of Fig.2. A first such situation we find in the case of the point P. In this point the connection between the segments B and C of the curve B-C is performed (see Fig.1). For such connection to be framed in the graphical representation of a continuous function (with no broken pieces), in this point P it must satisfy two conditions. The first one consists that in the point P there exists a commune geometric tangent to the curves B and C. In Algebra this means that the derivative of the function δ_C at the point P must have the same value with the derivative of the function δ_B at the respective point, thus for $\tau = 0.1$ we have:

$$(d\delta_B/d\tau)_{0.1} = (d\delta_C/d\tau)_{0.1}. \tag{6}$$

From (1) we have:

$$(d\delta_C/d\tau)_{0.1} = \tau(1.9044 - \tau^2)^{-0.5}. \tag{7}$$

Introducing $\tau = 0.1$ in (7) we obtain:

$$(d\delta_C/d\tau)_{0.1} = 0.0727. \tag{8}$$

On the other side, from (5) we have:

$$(d\delta_B/d\tau)_{0.1} = 4\alpha(0.1 + a - \tau)^3. \tag{9}$$

Introducing $\tau = 0.1$ in (9) we obtain:

$$(d\delta_B/d\tau)_{0.1} = 4\alpha a^3. \tag{10}$$

From the equalities (6), (8) and (10), we have :

$$\alpha a^3 = 0.0182. \tag{11}$$

The second condition is that function δ_{BC}, represented by the curve B continued with curve C, to be continuous at the point P is represented by the equality of the values of the two functions for $\tau = 0.1$, that is:

$$(d\delta_B)_{0.1} = (d\delta_C)_{0.1}. \tag{12}$$

Making in (1) and (5), $\tau = 0.1$ this equality becomes:

$$b = \alpha a^4 \tag{13}$$

and considering (11) we will have:

$$b = 0.0182a. \tag{14}$$

The third needed equation to calculate the values of α, a and b results from the application of equation (5) in the case of the point O which represents the origin of the coordinates system $\tau O\delta B$. Note that in this point we have $\tau = 0$ and $\delta B = 0$. Introducing these values in (5) we have:

$$\alpha = (0.584 + b)/(0.1 + a)^4. \tag{15}$$

Considering (14) we have:

$$\alpha = (0.584 + 0.0182a)/(0.1 + a)^4. \tag{16}$$

On the other side, from (11) we have:

$$\alpha = 0.0182/a^3. \tag{17}$$

The equations (16) and (17) contain the values of α and a, and solving this system of two equations we find the respective values. Evidently, in order to solve algebraically this system is very difficult and for this reason we will solve it graphically. Thus, if we give some values for a, than we get for α the values from the corresponding curves of these two equations. In the "medallion" of Fig.1 we represented these curves to see their form only. We denote with $\alpha(16)$ the curve corresponding to the equation (16) and with $\alpha(17)$ the curve corresponding to the equation (17). In order to make sure that we have a sufficient precision, these two curves were traced in a graph represented at a larger scale. At the intersection of these two curves we have the following values: $a = 0.0183; \alpha = 2982$. Introducing the value of a in (14) we obtain $b = 3.33 \times 10^{-4}$. We accept b = 0. With these values introduced in the equation (5) we obtain:

$$\delta_B = 0.584 - 2982(0.1183 - \tau)^4. \quad (18)$$

3. Using the "Switch" Coefficients in this Unique Mathematical Model

Looking at our results from above, we can keep in mind that the Universe Expansion can be simply represented using the algebraic language. The inconvenient presented in this representation is the fact that this model contains two equations, one the equation (18) which is valid for the no-dimensional time $\tau = (0\ to\ 0.1)$ and the other, the equation (1) which is valid for $\tau = (0.1\ to\ 1.0)$. This situation seems to be odd in such conditions in which there exists a perfect continuity between the function δ_B, equation (18) and the function δ_C, equation (1). In these conditions, the point P normally is part of a unique curve (B-C) which represents a unique function δ_{BC}. Next, we will describe the method used to eliminate this inconvenient. For this purpose we conceived some algebraic elements which we named "switch coefficients". We associated the word "coefficient" with the word "switch" to suggest that accepting the help of this coefficient we can switch from a situation to another different from the first. This can be done, considering the train trucks system, with the help of a switch. In our case, those two mentioned situations are expressed in a mathematical language. These are, as we will see, those functions which are geometrically represented by the curves B and C of Fig.1. For our purpose, these switch coefficients must cancel, one by one, one of the equations (1) and respectively (18), in function of the value of the variable τ with respect to a "threshold value" of this variable. In our case this threshold value is $\tau_P = 0.1$. For the values of τ smaller then τ_P, the equation (18) is valid and the equation (1) must be canceled and for the values $\tau > \tau_P$, the equation (1) is valid and the equation (18) must be canceled. We can invent the following switch coefficients (denoted by ν_B and ν_C) which guarantee the conditions mentioned before are given by the following equations:

$$\nu_B = 0.5(0.1 - \tau + |0.1 - \tau|)/|0.1 - \tau| \quad (19)$$

$$\nu_C = 0.5(\tau - 0.1 - |0.1 - \tau|)/|0.1 - \tau| \quad (20)$$

With these coefficients we obtain the following unique equation for the function δ_{BC} valid for the entire interval $\tau = (0 \rightarrow\ 1.0)$:

$$\delta_{BC} = \nu_B\delta_B + \nu_C\delta_C \quad (21)$$

Considering the equations (18) and (1) we have:

$$\delta_{BC} = \nu_B[0.584 - 2982(0.1183 - \tau)^4] + \nu_C[1.96 - (1.9044 - \tau^2)^{1/2}] \quad (22)$$

4. Conclusions

The Mathematical Models for Universe Expansion (UE) presented in (1) and (2), have some drawbacks regarding the values for the domain of τ for which are used and/or require complex calculations. At the beginning of the section 1 of this paper we exposed these drawbacks and also, we reminded the definitions of the no-dimensional measures δ and τ. These are the basic elements which intervene in the definition of the UE mathematical models (MM). The purpose of this paper consists in elaborating of a MM for UE in which all the drawbacks of the two models mentioned before will vanish. In order to achieve this purpose, firstly, we proceed to find another simple MM which can be applied for the domain of $\tau = (0\ to\ 0.1)$. Let remind that, for the domain $\tau = (0.1\ to\ 1.0)$ there exists a simple MM. This model was presented in [2] and it has as equation (1) as its mathematical expression. Its graphical expression is represented by the curves C of Fig.1 and Fig.2. For $\tau = (0\ to\ 0.1)$ the desired MM was elaborated and described in the section 1 above. It is represented by the equation (18) and it has its graphical correspondent is in the curves B of Fig.1 and Fig.2. In this way we shaped two distinct MM for UE. The first (δ_B) is valid for the domain $\tau = (0\ to\ 0.1)$ and the second (δ_C) is valid for the domain $\tau = (0.1\ to\ 1.0)$. In order to obtain a MM for the entire domain, $\tau = (0\ to\ 1.0)$ we introduced some mathematical elements which we named "switch coefficients". These (ν_B and ν_C) are given in the equations (19) and (20) in function of τ and a "threshold value" of this variable which in our case is $\tau_P = 0.1$. From the equations (19) and (20) it follows that for $\tau < 0.1$ we have $\nu_B = 1$ and $\nu_C = 0$ and for $\tau > 0.1$ we have $\nu_B = 0$ and $\nu_C = 1$. With these conditions MM needed is that represented by the equation (22). Its graphical representation in Fig.1 is the continuous curve B, followed without interruptions by the curve C. This model is simple, if we need to calculate δ_{BC} in function of τ and in the same time valid for the entire domain of the no-dimensional time $\tau = (0\ to\ 1.0)$.

References

[1] Baica M., Cardu M., *A Mathematical Model for the Universe Expansion*, Transylv. J. Math. Mech., 8(2016), No. 1, Edyro Press Publisher, 15–20.

[2] Baica M., Cardu M., *A simplifier Mathematical Model of the Universe Expansion and an Estimation of its Life Time Duration*, Transylv. J. Math. Mech., 8(2016), No. 2, Edyro Press Publisher, 123–127.

[3] Musatoiu S., *The Study of Graphical Functions Variation (in Romanian)*, Matrix Rom Publishing House, Bucharest, 2004.

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