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ON WEIGHTED OSTROWSKI GRÜSS INEQUALITY WITH APPLICATIONS

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Abstract. The purpose of present article is to obtain weighted Ostrowski Grüss type inequality for differentiable functions by using weighted Korkine's identity. We also discussed its applications into probability density function and numerical quadrature rules.

1. Introduction

Inequalities play significant role in the field of functional analysis, optimization theory, numerical analysis, probability theory and calculus. In numerical analysis, inequalities help us to find out the best possible bounds. The present article is devoted to an obvious extension towards the Ostrowski Grüss inequality in terms of weights.

Grüss inequality is a relation between the integral of the product of the two functions and the product of the integral of the two functions.

The Grüss inequality [5] is stated as:

Proposition 1. *Let ϕ and φ be two functions defined and integrable on $[a_0, a_1]$. Further, let the Čebyšev functional be defined as*

$$T(\phi,\varphi) = \frac{1}{a_1-a_0}\int_{a_0}^{a_1}\phi(\xi)\varphi(\xi)d\xi - \left(\frac{1}{a_1-a_0}\int_{a_0}^{a_1}\phi(\xi)d\xi\right)\left(\frac{1}{a_1-a_0}\int_{a_0}^{a_1}\varphi(\xi)d\xi\right)$$

$$m \leq \phi(\xi) \leq M, \quad \gamma \leq \varphi(\xi) \leq \Gamma,$$

for each $\xi \in [a_0, a_1]$, where ν, μ, γ, Γ are given real constants. Then,

$$|T(\phi,\varphi)| \leq \frac{1}{4}(M-m)(\Gamma-\gamma),$$

where the constant $\frac{1}{4}$ is the best possible.

In 1997, by using Grüss inequality, S.S. Dragomir and S. Wang [3] proved the following Ostrowski Grüss type integral inequality:

Proposition 2. *Let $\phi : I \to \mathbb{R}$, where $I \subseteq \mathbb{R}$ is an interval, be a mapping differentiable in the interior I^0 of I and let $a_0, a_1 \in I^0$ with $a_0 < a_1$. If $\nu \leq \phi'(\xi) \leq \mu$, $\xi \in [a_0, a_1]$ for some constants $\nu, \mu \in \mathbb{R}$, then*

$$\left|\phi(\xi) - \frac{1}{a_1-a_0}\int_{a_0}^{a_1}\phi(s)ds - \frac{\phi(a_1)-\phi(a_0)}{a_1-a_0}\left(\xi - \frac{a_0+a_1}{2}\right)\right| \leq \frac{1}{4}(a_1-a_0)(\mu-\nu) \tag{1}$$

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for all $\xi \in [a_0, a_1]$.

The above inequality creates a relation between Ostrowski inequality [9] and the Grüss inequality [8].

In 2000, Proposition 2 was improved by M. Matić, J. E. Pečarić and N. Ujević [7].

Proposition 3. *Let the assumptions of Proposition 2 be true. Then we have the following inequality*

$$\left|\phi(\xi) - \frac{1}{a_1 - a_0}\int_{a_0}^{a_1} \phi(s)ds - \frac{\phi(a_1) - \phi(a_0)}{a_1 - a_0}\left(\xi - \frac{a_0 + a_1}{2}\right)\right| \leq \frac{1}{4\sqrt{3}}(\mu - \nu)(a_1 - a_0) \tag{2}$$

for all $\xi \in [a_0, a_1]$.

In the same year, N.S. Barnett, S.S. Dragomir and A. Sofo [1] further improved that inequality (2), which states that:

Proposition 4. *Let* $\phi : I \to \mathbb{R}$ *be an absolutely continuous function whose derivative* $\phi' \in L_2[a_0, a_1]$*, if* $\nu \leq \phi'(\xi) \leq \mu$*,* $\xi \in [a_0, a_1]$ *for some constants* $\nu, \mu \in \mathbb{R}$*. Then we have the following inequality*

$$\begin{aligned} &\left|\phi(\xi) - \frac{1}{a_1 - a_0}\int_{a_0}^{a_1} \phi(s)ds - \frac{\phi(a_1) - \phi(a_0)}{a_1 - a_0}\left(\xi - \frac{a_0 + a_1}{2}\right)\right| \\ &\leq \frac{(a_1 - a_0)}{2\sqrt{3}}\left[\frac{1}{a_1 - a_0}\|\phi'\|_2^2 - \left(\frac{\phi(a_1) - \phi(a_0)}{a_1 - a_0}\right)^2\right]^{\frac{1}{2}} \\ &\leq \frac{1}{4\sqrt{3}}(\mu - \nu)(a_1 - a_0). \end{aligned} \tag{3}$$

In the present article, we generalized the inequality (3) for differentiable function in terms of weight, by using weighted Peano kernel as defined in [6]. Our first section is based on introduction and preliminaries. In the second section, we discuss weighted Grüss inequality by using the technique of weighted Korkine's identity. In the third section, we then applied it into probability density function. Our last section is based on some applications of numerical quadrature rules.

2. Main results

Here we need two lemmas, by using the following weighted Korkine's identity (4) and weighted Grüss inequality (5) as mentioned in [2], we will prove our main result.

Lemma 1. *Let* $\omega, \phi, \varphi : [a_0, a_1] \to \mathbb{R}$ *be the measurable mapping for which the integrals involved in the following identity exist and finite, then*

$$\begin{aligned} &\int_{a_0}^{a_1} \omega(s)ds \int_{a_0}^{a_1} \omega(s)\phi(s)\varphi(s)ds - \int_{a_0}^{a_1} \omega(s)\phi(s)ds \int_{a_0}^{a_1} \omega(s)\varphi(s)ds \\ &= \frac{1}{2}\int_{a_0}^{a_1}\int_{a_0}^{a_1} \omega(s)\omega(t)\left(\phi(s) - \phi(t)\right)\left(\varphi(s) - \varphi(t)\right)dsdt. \end{aligned} \tag{4}$$

Lemma 2. *Let the assumptions of Lemma 1 be valid, then we have the following Ostrowski Grüss inequality*

$$0 \leq \int_{a_0}^{a_1} \omega(s)\phi^2(s)ds - \left(\int_{a_0}^{a_1} \omega(s)\phi(s)ds\right)^2 \leq \frac{1}{4}(M - m)^2 \tag{5}$$

where $m \leq \phi(s) \leq M$ *a.e on* $[a_0, a_1]$.

Theorem 1. *Let the assumptions of Proposition 4 be valid. Then we have the following inequality*

$$\begin{aligned}
&\left|\phi(\xi)-\left(\xi-\int_{a_0}^{a_1}\omega(s)sds\right)\left(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\right)-\int_{a_0}^{a_1}\omega(s)\phi(s)ds\right| \\
\leq&\left[\int_{a_0}^{a_1}\frac{P_\omega^2(\xi,s)ds}{\omega(s)}-\left(\int_{a_0}^{a_1}P_\omega(\xi,s)ds\right)^2\right]^{\frac{1}{2}} \\
&\times\left[\int_{a_0}^{a_1}\omega(s)[\phi'(s)]^2ds-\left(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\right)^2\right]^{\frac{1}{2}} \\
\leq&\frac{1}{2}(\mu-\nu)H_\omega(\xi,s), \qquad (6)
\end{aligned}$$

where

$$H_\omega(\xi,s)=\int_{a_0}^{a_1}\frac{P_\omega^2(\xi,s)ds}{\omega(s)}-\left(\int_{a_0}^{a_1}P_\omega(\xi,s)ds\right)^2$$

and $\omega:[a_0,a_1]\to[0,\infty)$ *is some probability density function satisfying*

$$\int_{a_0}^{a_1}\omega(s)ds=1$$

for all $\xi\in[a_0,a_1]$.

Proof. We have the kernel as defined in [6], $P_\omega(\xi,s):[a_0,a_1]^2\to\mathbb{R}$

$$P_\omega(\xi,s)=\begin{cases}\displaystyle\int_{a_0}^{s}\omega(u)du, & \text{if} \quad s\in[a_0,\xi],\\ \displaystyle\int_{a_1}^{s}\omega(u)du, & \text{if} \quad s\in(\xi,a_1].\end{cases}$$

From (4), we get the Korkine's identity in the form of

$$\begin{aligned}
&\int_{a_0}^{a_1}P_\omega(\xi,s)\phi'(s)ds-\int_{a_0}^{a_1}P_\omega(\xi,s)ds\int_{a_0}^{a_1}\omega(s)\phi'(s)ds \\
&=\int_{a_0}^{a_1}\int_{a_0}^{a_1}\omega(s)\omega(t)\left(\frac{P_\omega(\xi,s)}{\omega(s)}-\frac{P_\omega(\xi,t)}{\omega(t)}\right)(\phi'(s)-\phi'(t))\,dsdt. \qquad (7)
\end{aligned}$$

From [6], we have

$$\int_{a_0}^{a_1}P_\omega(\xi,s)\phi'(s)ds \;=\; \phi(\xi)-\int_{a_0}^{a_1}\omega(s)\phi(s)ds \qquad (8)$$

and

$$\int_{a_0}^{a_1}P_\omega(\xi,s)ds=\xi-\int_{a_0}^{a_1}\omega(s)sds. \qquad (9)$$

By putting (8) and (9) in (7), we get

$$\begin{aligned}
&\phi(\xi)-\left(\xi-\int_{a_0}^{a_1}\omega(s)sds\right)\left(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\right)-\int_{a_0}^{a_1}\omega(s)\phi(s)ds \\
&=\int_{a_0}^{a_1}\int_{a_0}^{a_1}\omega(s)\omega(t)\left(\frac{P_\omega(\xi,s)}{\omega(s)}-\frac{P_\omega(\xi,t)}{\omega(t)}\right)(\phi'(s)-\phi'(t))\,dsdt \qquad (10)
\end{aligned}$$

$\forall\ \xi \in [a_0, a_1]$. Using Cauchy Schwartz inequality for double integrals, we get

$$\begin{aligned} &\left|\frac{1}{2}\int_{a_0}^{a_1}\int_{a_0}^{a_1}\omega(s)\omega(t)\left(\frac{P_\omega(\xi,s)}{\omega(s)}-\frac{P_\omega(\xi,t)}{\omega(t)}\right)(\phi'(s)-\phi'(t))\,dsdt\right| \\ \leq\ &\left(\frac{1}{2}\int_{a_0}^{a_1}\int_{a_0}^{a_1}\omega(s)\omega(t)\left(\frac{P_\omega(\xi,s)}{\omega(s)}-\frac{P_\omega(\xi,t)}{\omega(t)}\right)^2 dsdt\right)^{\frac{1}{2}} \\ &\times\left(\frac{1}{2}\int_{a_0}^{a_1}\int_{a_0}^{a_1}\omega(s)\omega(t)\left(\phi'(s)-\phi'(t)\right)^2 dsdt\right)^{\frac{1}{2}}. \end{aligned} \tag{11}$$

By using (7), we get the following identities

$$\begin{aligned} &\frac{1}{2}\int_{a_0}^{a_1}\int_{a_0}^{a_1}\omega(s)\omega(t)\left(\frac{P_\omega(\xi,s)}{\omega(s)}-\frac{P_\omega(\xi,t)}{\omega(t)}\right)^2 dsdt \\ &\qquad = \int_{a_0}^{a_1}\frac{P_\omega^2(\xi,s)ds}{\omega(s)}-\left(\int_{a_0}^{a_1}P_\omega(\xi,s)ds\right)^2 \end{aligned} \tag{12}$$

and

$$\begin{aligned} &\frac{1}{2}\int_{a_0}^{a_1}\int_{a_0}^{a_1}\omega(s)\omega(t)\left(\phi'(s)-\phi'(t)\right)^2 dsdt \\ &\qquad = \int_{a_0}^{a_1}\omega(s)[\phi'(s)]^2 ds-\Big(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\Big)^2. \end{aligned} \tag{13}$$

Using weighted Ostrowski Grüss inequality (5), if $\nu \leq \phi'(s) \leq \mu$ and $s \in (a_0, a_1)$, we get

$$0 \leq \int_{a_0}^{a_1}\omega(s)\big(\phi'(s)\big)^2 ds-\Big(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\Big)^2 \leq \frac{1}{4}(\mu-\nu)^2. \tag{14}$$

Using (10)–(14), we obtain

$$\begin{aligned} &\left|\phi(\xi)-\left(\xi-\int_{a_0}^{a_1}\omega(s)sds\right)\left(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\right)-\int_{a_0}^{a_1}\omega(s)\phi(s)ds\right| \\ \leq\ &\left[\int_{a_0}^{a_1}\frac{P_\omega^2(\xi,s)ds}{\omega(s)}-\left(\int_{a_0}^{a_1}P_\omega(\xi,s)ds\right)^2\right]^{\frac{1}{2}} \\ &\times\left[\int_{a_0}^{a_1}\omega(s)[\phi'(s)]^2 ds-\left(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\right)^2\right]^{\frac{1}{2}} \\ \leq\ &\frac{1}{2}(\mu-\nu)\left[\int_{a_0}^{a_1}\frac{P_\omega^2(\xi,s)ds}{\omega(s)}-\left(\int_{a_0}^{a_1}P_\omega(\xi,s)ds\right)^2\right] \\ =\ &\frac{1}{2}(\mu-\nu)H_\omega(\xi,s) \end{aligned}$$

which proves our result (6). □

We can define some special cases of (6).

Remark 1. *If we put* $\omega(s) \equiv \frac{1}{a_1-a_0}$ *in* (6)*, then we get the special result* (3) *of* [1].

Corollary 1. *If we put $\xi = \frac{a_0+a_1}{2}$ in (6), then we get the following midpoint inequality*

$$\left|\phi\left(\frac{a_0+a_1}{2}\right) - \left(\frac{a_0+a_1}{2} - \int_{a_0}^{a_1}\omega(s)sds\right)\left(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\right) - \int_{a_0}^{a_1}\omega(s)\phi(s)ds\right|$$
$$\leq \left[\int_{a_0}^{a_1}\frac{P_\omega^2\left(\frac{a_0+a_1}{2},s\right)ds}{\omega(s)} - \left(\int_{a_0}^{a_1}P_\omega\left(\frac{a_0+a_1}{2},s\right)ds\right)^2\right]^{\frac{1}{2}}$$
$$\times\left[\int_{a_0}^{a_1}\omega(s)[\phi'(s)]^2ds - \left(\int_{a_0}^{a_1}\omega(s)\phi'(s)ds\right)^2\right]^{\frac{1}{2}}$$
$$\leq \frac{1}{2}(\mu-\nu)H_\omega\left(\frac{a_0+a_1}{2},s\right). \tag{15}$$

Remark 2. *If we put $\omega(s) = \frac{1}{a_1-a_0}$ in (15), then we get the following midpoint inequality*

$$\left|\phi\left(\frac{a_0+a_1}{2}\right) - \frac{1}{a_1-a_0}\int_{a_0}^{a_1}\phi(s)ds\right|$$
$$\leq \frac{(a_1-a_0)}{2\sqrt{3}}\left[\frac{1}{a_1-a_0}\|\phi'\|_2^2 - \left(\frac{\phi(a_1)-\phi(a_0)}{a_1-a_0}\right)^2\right]^{\frac{1}{2}}$$
$$\leq \frac{1}{4\sqrt{3}}(\mu-\nu)(a_1-a_0).$$

Remark 3. *If we put $\xi = a_0$ or $\xi = a_1$ and $\omega(s) \equiv \frac{1}{a_1-a_0}$ in (6), then we get the trapezoidal inequality which is independent the value of h*

$$\left|\frac{\phi(a_0)+\phi(a_1)}{2} - \frac{1}{a_1-a_0}\int_{a_0}^{a_1}\phi(s)ds\right|$$
$$\leq \frac{a_1-a_0}{2\sqrt{3}}\left[\frac{1}{a_1-a_0}\|\phi'\|_2 - \left(\frac{\phi(a_1)-\phi(a_0)}{a_1-a_0}\right)^2\right]^{\frac{1}{2}}$$
$$\leq \frac{(a_1-a_0)(\mu-\nu)}{4\sqrt{3}}. \tag{16}$$

In the next section, we are going to discuss some applications of our main result for Probability density functions.

3. Application for Probability Density Functions

Let X be a continuous random variable having the probability density function $\phi : [a_0, a_1] \to \mathbb{R}_+$ and the cumulative distribution function $\Phi : [a_0, a_1] \to [0, 1]$, i.e.,

$$\Phi(\xi) = \int_{a_0}^{\xi}\phi(s)ds, \quad \xi \in [\alpha_0, \alpha_1] \subset [a_0, a_1],$$

$$E(X) = \int_{a_0}^{a_1}s\phi(s)ds,$$

and weighted expectation would be

$$E_\omega(X) = \int_{a_0}^{a_1}\omega(s)s\phi(s)ds$$

on the interval $[a_0, a_1]$. Then we may have the following theorem.

Theorem 2. *Let the assumptions of Theorem 1 be valid and if probability density function belongs to $L_2[a_0, a_1]$ space, then we get the inequality*

$$\begin{aligned}&\Bigg|\Phi(\xi) - \omega(a_1)a_1 - E_w(X) - \left(\xi - \int_{a_0}^{a_1} \omega(s)sds\right)\\&\times\left(\omega(a_1) - \int_{a_0}^{a_1} \omega'(s)\Phi(s)ds\right) - \int_{a_0}^{a_1} \omega'(s)s\Phi(s)ds\Bigg|\\&\leq \frac{1}{2}(\mu - \nu)H_\omega(\xi, s) \qquad (17)\end{aligned}$$

for all $\xi \in [\alpha_0, \alpha_1]$.

Proof. Put $\phi = \Phi$ in (6) and by using these two identities mention below, we get (17),

$$\int_{a_0}^{a_1} \omega(s)\Phi(s)ds = a_1\omega(a_1) - E_\omega(X) - \int_{a_0}^{a_1} \omega'(s)s\Phi(s)ds$$

and

$$\int_{a_0}^{a_1} \omega(s)\Phi'(s)ds = \omega(a_1) - \int_{a_0}^{a_1} \omega'(s)\Phi(s)ds.$$

□

Remark 4. *Under the assumptions of Theorem 2, if we substitute $\omega(s) \equiv \frac{1}{a_1 - a_0}$ in (17), then we get the following inequality*

$$\begin{aligned}&\left|\Phi(\xi) - \frac{1}{a_1 - a_0}\left(\xi - \frac{a_0 + a_1}{2}\right) - \frac{a_1 - E(X)}{a_1 - a_0}\right|\\\leq\ &\frac{(a_1 - a_0)}{2\sqrt{3}}\left[\frac{1}{a_1 - a_0}\|\phi'\|_2^2 - \left(\frac{\phi(a_1) - \phi(a_0)}{a_1 - a_0}\right)^2\right]^{\frac{1}{2}}\\\leq\ &\frac{1}{4\sqrt{3}}(\mu - \nu)(a_1 - a_0).\end{aligned}$$

4. Applications to Numerical Quadrature Rules

Now, we are going to discuss some applications in numerical quadrature rules, which can be used to get some bounds.
Let $I_n : a_0 = z_0 < z_1 < \ldots < z_n = a_1$ be a partition of the interval $[a_0, a_1]$ and let $\Delta z_k = z_{k+1} - z_k, k \in \{0, 1, 2, \ldots, n-1\}$. Then

$$\int_{z_k}^{z_{k+1}} \omega(s)\phi(s)ds = Q_n(I_n, \phi, \omega) + R_n(I_n, \phi, \omega) \qquad (18)$$

where $Q_n(I_n, \phi, \omega)$ is a quadrature formula, define as

$$\begin{aligned}Q_n(I_n, \phi, \omega) &= \sum_{k=0}^{n-1}\Bigg[\phi(\xi) - \left(\xi_k - \int_{z_k}^{z_{k+1}} \omega(s)sds\right)\\&\quad\times\left(\int_{z_k}^{z_{k+1}} \omega(s)\phi'(s)ds\right)\Bigg] \qquad (19)\end{aligned}$$

for all $\xi_k \in [z_k, z_{k+1}]$.

Theorem 3. *Let ϕ as be defined in Theorem 1. Then* (6) *holds where $Q_n(I_n,\phi,\omega)$ is given by formula* (19) *and the remainder $R_n(I_n,\phi,\omega)$ satisfies the estimates*

$$|R_n(I_n,\phi,\omega)| \leq \sum_{k=0}^{n-1} \frac{(\mu-\nu)}{2} H_\omega(\xi_k, s) \tag{20}$$

for all $\xi_k \in [z_k, z_{k+1}]$.

Proof. Applying inequality (6) on the intervals $[z_k, z_{k+1}]$, we can state that

$$\begin{aligned} R_k(I_k,\phi,\omega) &= \int_{z_k}^{z_{k+1}} \omega(s)\phi(s)ds - \phi(\xi_k) \\ &\quad + \left(\xi_k - \int_{z_k}^{z_{k+1}} \omega(s)sds\right) \times \left(\int_{z_k}^{z_{k+1}} \omega(s)\phi'(s)ds\right). \end{aligned}$$

Now, we sum the inequalities presented above over k from 0 to $n-1$. This gives

$$\begin{aligned} R_n(I_k,\phi,\omega) &= \sum_{k=0}^{n-1} \int_{z_k}^{z_{k+1}} \omega(s)\phi(s)ds - \sum_{k=0}^{n-1} \Big[\phi(\xi_k) \\ &\quad - \left(\xi_k - \int_{z_k}^{z_{k+1}} \omega(s)sds\right) \times \left(\int_{z_k}^{z_{k+1}} \omega(s)\phi'(s)ds\right)\Big]. \end{aligned}$$

Applying absolute property on the above identity, we get

$$\begin{aligned} |R_n(I_k,\phi,\omega)| &= \Bigg|\sum_{k=0}^{n-1} \int_{z_k}^{z_{k+1}} \omega(s)\phi(s)ds - \sum_{k=0}^{n-1} \Big[\phi(\xi_k) \\ &\quad - \left(\xi_k - \int_{z_k}^{z_{k+1}} \omega(s)sds\right) \times \left(\int_{z_k}^{z_{k+1}} \omega(s)\phi'(s)ds\right)\Big]\Bigg| \\ &\leq \frac{1}{2}(\mu-\nu)H_\omega(\xi_k, s). \end{aligned}$$

□

If we choose $\omega(s) \equiv \frac{1}{\Delta z_k}$ in (18) and (19), then we get the identity

$$\frac{1}{\Delta z_k} \int_{z_k}^{z_{k+1}} \phi(s)ds = Q_n(I_n,\phi) + R_n(I_n,\phi) \tag{21}$$

and the quadrature formula becomes

$$Q_n(I_n,\phi) = \sum_{k=0}^{n-1} \left[\phi(\xi_k) - \frac{\phi(z_{k+1}) - \phi(z_k)}{\Delta z_k}\left(\xi_k - \frac{z_k + z_{k+1}}{2}\right)\right]. \tag{22}$$

Remark 5. *If we choose $\omega(s) \equiv \frac{1}{\Delta z_k}$ in* (20)*, we archive the identity* (21) *and the quadrature formula $Q_n(I_n,\phi)$* (22)*, then remainder $R_n(I_n,\phi)$ satisfies the estimates*

$$|R_n(I_n,\phi)| \leq \sum_{k=0}^{n-1} \frac{(\mu-\nu)\Delta z_k^2}{8\sqrt{3}}$$

where

$$\begin{aligned} R_n(I_n,\phi) &= \sum_{k=0}^{n-1} \Bigg[\frac{1}{\Delta z_k} \int_{z_k}^{z_{k+1}} \phi(s)ds \\ &\quad - \left\{\phi(\xi_k) - \frac{\phi(z_{k+1}) - \phi(z_k)}{\Delta z_k}\left(\xi_k - \frac{z_k + z_{k+1}}{2}\right)\right\}\Bigg]. \end{aligned}$$

5. Conclusion

In present article we have generalized the Ostrowski Grüss inequality. By introducing the weighted kernel as defined in [6], we have obtained generalization of Ostrowski Grüss integral inequality for first differentiable functions in terms of weights. By using appropriate substitution we get different previous published results. At the end, we have also discussed some applications for probability density functions and numerical quadrature rules.

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