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SOME PROPERTIES OF CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS ASSOCIATED WITH GENERALIZED DIFFERENTIAL OPERATOR INVOLVING MITTAG-LEFFLER FUNCTION

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Abstract. A subclass in the open unit disc of analytic functions is introduced in this paper. This subclass $\mathcal{S}_{\vartheta}^{m,\delta}(\lambda,\alpha,\beta,A,B)$ is mainly defined by generalized differential Operator. Some interesting properties of this class are also obtained.

1. Introduction

We begin by letting $\mathbb{U}=\{z\in\mathbb{C}:|z|<1\}$ be the open unit disk of the complex plane and $\mathcal{H}=\mathcal{H}(\mathbb{U})$ denote the class of analytic functions. For $a\in\mathbb{C}$ and $n\in\mathbb{N}$, let $\mathcal{H}[a,n]$ be the subclass of $\mathcal{H}$ consisting of the functions of the form

$$f(z)=a+a_nz^n+a_{n+1}z^{n+1}+......,\qquad (a\in\mathbb{C}).$$

And, let $\mathcal{A}$ be the subclass of $\mathcal{H}$ consisting of functions of the form

$$f(z)=z+\sum_{k=2}^{\infty}a_kz^k\qquad a_k\geq 1. \tag{1}$$

A function f belonging to the class $\mathcal{A}$ is said to be convex of order α $(0\leq\alpha<1)$ if and only if $\mathcal{R}e\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\alpha$, $z\in\mathbb{U}$. This class is denoted by $\mathcal{C}(\alpha)$. On the other hand, a function $f\in\mathcal{A}$ is said to be starlike of order α $(0\leq\alpha<1)$ if and only if $\mathcal{R}e\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha$, $z\in\mathbb{U}$. This class is denoted by $\mathcal{S}^*(\alpha)$.

The Hadamard product for two analytic functions f defined in (1) and

$$g(z)=z+\sum_{k=2}^{\infty}b_kz^k\qquad b_k\geq 1,$$

is given by

$$f(z)*g(z)=z+\sum_{k=2}^{\infty}a_kb_kz^k.$$

Given two analytic functions f and g in $\mathbb{U}$, the subordination between them is written as $f\prec g$ or $f(z)\prec g(z)$, In addition, we say $f(z)$ is subordinate to $g(z)$ if there is a Schwarz function w with $w(z)=0$, $|w(z)|<1$, $(z\in\mathbb{U})$ such that $f(z)=g(w(z))$ for all $z\in\mathbb{U}$. Furthermore, if $g(z)$ is univalent in $\mathbb{U}$, then $f\prec g$ if and only if $f(0)=g(0)$ and $f(\mathbb{U})\subseteq g(\mathbb{U})$.

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The following defines the familiar Mittag-Leffler function $E_\alpha(z)$ introduced by Mittag-Leffler [7] and [8] and its generalization $E_{\alpha,\beta}(z)$ introduced by Wiman [15]

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)},$$

and

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)},$$

where $\alpha, \beta \in \mathbb{C}$, $Re(\alpha) > 0$ and $Re(\beta) > 0$.

As a result, a lot of useful work have been made by many researchers in attempt to explain Mittag-Leffler function and its generalization, see for example [13], [2], [9], [11], and [14].

Now, we define the function $Q_{\alpha,\beta}(z)$ by

$$\begin{aligned} Q_{\alpha,\beta}(z) &= z\Gamma(\beta)E_{\alpha,\beta}(z) \\ &= z + \sum_{k=2}^{\infty} \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} z^k. \end{aligned}$$

Corresponding to $Q_{\alpha,\beta}(z)$, Elhaddad et al.[3] defined the differential operator $D_\lambda^m(\alpha,\beta)f : \mathcal{A} \longrightarrow \mathcal{A}$ as follows :

$$D_\lambda^m(\alpha,\beta)f(z) = z + \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k z^k, \tag{2}$$

where $m \in \mathbb{N}_0 = \mathbb{N} \bigcup \{0\}, \lambda \geq 0$.

We can easily verify from (2) that

$$D_\lambda^{m+1}(\alpha,\beta)f(z) = (1-\lambda)D_\lambda^m(\alpha,\beta)f(z) + \lambda z(D_\lambda^m(\alpha,\beta)f(z))' \tag{3}$$

Note that

- for $\alpha = 0$ and $\beta = 1$ we get Al-Oboudi operator [1];
- for $\alpha = 0$, $\beta = 1$, $\lambda = 1$ we get Sălăgean operator [10];
- for $m = 0$ we get $\mathbb{E}_{\alpha,\beta}(z)$ [13].

Next, as a result of full utilization of the differential operator $D_\lambda^m(\alpha,\beta)f(z)$ and the principle of subordination, we define and study the class $\mathcal{S}_\vartheta^{m,\delta}(\lambda,\alpha,\beta,A,B)$ as follows :

Definition 1. *A function f belonging to the class $\mathcal{A}$ is said to be in the class $\mathcal{S}_\vartheta^{m,\delta}(\lambda,\alpha,\beta, A,B)$ if it satisfies the following subordination condition*

$$(1-\vartheta)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta + \vartheta\left(\frac{D_\lambda^{m+1}(\alpha,\beta)f(z)}{D_\lambda^m(\alpha,\beta)f(z)}\right)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta \prec \frac{1+Az}{1+Bz},$$

where $\vartheta \in \mathbb{C}, \mathcal{R}e(\delta) > 0, m \in \mathbb{N}_0 = \mathbb{N}\bigcup\{0\}, \lambda \geq 0, -1 \leq B \leq 1$ and $A \neq B \in \mathbb{N}_0$.

Note that, if we set $m = 0, \lambda = 1, \alpha = 0$ and $\beta = 1$ in the class $\mathcal{S}_\vartheta^{m,\delta}(\lambda,\alpha,\beta,A,B)$, then we get the class studied by Liu [4].

In order to prove and validate above results, following preliminary results are required.

2. Preliminary Results

Lemma 1. *(see [5]). Let the function h be analytic and univalent (convex) in $\mathbb{U}$ with $h(0) = 1$. Suppose also that the function φ given by*

$$\varphi(z) = 1 + c_n z^n + c_{n+1} z^{n+1} + ...,$$

is analytic in $\mathbb{U}$. If

$$\varphi(z) + \frac{z\varphi^{'}(z)}{\varsigma} \prec h(z) \quad (\mathcal{R}e(\varsigma) \geq 0; \varsigma \neq 0, z \in \mathbb{U}),$$

then

$$\varphi(z) \prec \phi(z) = \frac{\varsigma}{n} z^{\frac{-\varsigma}{n}} \int_0^z t^{\frac{\varsigma}{n}-1} h(t)dt \prec h(z) \quad (z \in \mathbb{U}),$$

and $\phi(z)$ is the best dominant.

Lemma 2. *(see [12]). Let $q(z)$ be a convex univalent function in $\mathbb{U}$ and let $\sigma, \eta \in \mathbb{C}$ with*

$$\mathcal{R}e\left\{1 + \frac{zq^{''}(z)}{q^{'}(z)}\right\} > max\left\{0, -\mathcal{R}e\left(\frac{\sigma}{\eta}\right)\right\}.$$

If the function p is analytic in $\mathbb{U}$ and

$$\sigma p(z) + \eta z p^{'}(z) \prec \sigma q(z) + \eta z q^{'}(z),$$

then

$$p(z) \prec q(z),$$

and $q(z)$ is the best dominant.

Lemma 3. *(see [6]). Let q be convex univalent in $\mathbb{U}$ and $k \in \mathbb{C}$. Further assume that $\mathcal{R}e(k) > 0$, if*

$$p(z) \in \mathcal{H}[q(0), 1] \cap Q,$$

and $p(z) + kzp^{'}(z)$ is univalent in $\mathbb{U}$, then

$$q(z) + kzq^{'}(z) \prec p(z) + kzp^{'}(z)$$

Implies $q(z) \prec p(z)$ and $q(z)$ is the best subdominant

3. Main Result

In what follows we focus to study a set of interesting characteristics of the class $\mathcal{S}_{\vartheta}^{m,\delta}(\lambda, \alpha, \beta, A, B)$

Theorem 1. *Let $f \in \mathcal{S}_{\vartheta}^{m,\delta}(\lambda, \alpha, \beta, A, B)$. Then*

$$\left(\frac{D_\lambda^m(\alpha, \beta)f(z)}{z}\right)^\delta \prec \frac{\delta}{\lambda n \vartheta} \int_0^1 \frac{1 + Azu}{1 + Bzu} u^{\frac{\delta}{\lambda n \vartheta}-1} du \prec \frac{1 + Az}{1 + Bz},$$

where $\vartheta \in \mathbb{C}, \mathcal{R}e(\delta) > 0, m \in \mathbb{N}_0 = \mathbb{N} \bigcup \{0\}, \lambda \geq 0, -1 \leq B \leq 1$ and $A \neq B \in \mathbb{N}_0$.

Proof. Define the function $p(z)$ by

$$p(z) = \left(\frac{D_\lambda^m(\alpha, \beta)f(z)}{z}\right)^\delta \quad (z \in \mathbb{U}). \tag{4}$$

Then the function $p(z)$ is analytic in $\mathbb{U}$ with $p(0) = 1$. Therefore, differentiating (4) logarithmically with respect to z, and using the identity (3), we get

$$(1-\vartheta)\left(\frac{D_{\lambda}^{m}(\alpha,\beta)f(z)}{z}\right)^{\delta}+\vartheta\left(\frac{D_{\lambda}^{m+1}(\alpha,\beta)f(z)}{D_{\lambda}^{m}(\alpha,\beta)f(z)}\right)\left(\frac{D_{\lambda}^{m}(\alpha,\beta)f(z)}{z}\right)^{\delta} \prec p(z)+\frac{\lambda\vartheta}{\delta}zp^{'}(z)\prec\frac{1+Az}{1+Bz}\quad(z\in\mathbb{U}). \tag{5}$$

By applying Lemma 1 in the last equation, we have

$$\left(\frac{D_{\lambda}^{m}(\alpha,\beta)f(z)}{z}\right)^{\delta}\prec\frac{\delta}{\lambda n\vartheta}z^{-\frac{\delta}{\lambda n\vartheta}}\int_{0}^{z}t^{\frac{\delta}{\lambda n\vartheta}-1}\frac{1+At}{1+Bt}du = \frac{\varsigma}{n}\int_{0}^{1}u^{\frac{\varsigma}{n}-1}\frac{1+Azu}{1+Bzu}du\prec\frac{1+Az}{1+Bz}\quad(z\in\mathbb{U}),$$

where $\varsigma=\dfrac{\delta}{\lambda\vartheta}$.

The proof of Theorem 1 is complete. □

Theorem 2. *Let $q(z)$ be univalent in $\mathbb{U}$. Suppose also that $q(z)$ satisfies*

$$\mathcal{R}e\left\{1+\frac{zq^{''}(z)}{q^{'}(z)}\right\}>max\left\{0,-\mathcal{R}e\left(\frac{\delta}{\lambda\vartheta}\right)\right\}. \tag{6}$$

If $f(z)\in\mathcal{A}$ satisfying the following subordination

$$(1-\vartheta)\left(\frac{D_{\lambda}^{m}(\alpha,\beta)f(z)}{z}\right)^{\delta}+\vartheta\left(\frac{D_{\lambda}^{m+1}(\alpha,\beta)f(z)}{D_{\lambda}^{m}(\alpha,\beta)f(z)}\right)\left(\frac{D_{\lambda}^{m}(\alpha,\beta)f(z)}{z}\right)^{\delta} \prec q(z)+\frac{\lambda\vartheta}{\delta}zq^{'}(z), \tag{7}$$

then

$$\left(\frac{D_{\lambda}^{m}(\alpha,\beta)f(z)}{z}\right)^{\delta}\prec q(z),$$

and $q(z)$ is the best dominant.

Proof. Let $p(z)$ be defined by (4). We know that (5) is true. Combining (5) and (7), we see that

$$p(z)+\frac{\lambda\vartheta}{\delta}zp^{'}(z)\prec q(z)+\frac{\lambda\vartheta}{\delta}zq^{'}(z). \tag{8}$$

By using Lemma 2 and (8), we get the assertion of Theorem 2. □

Taking $q(z)=\dfrac{1+Az}{1+Bz}$ in Theorem 2, we get the following result.

Corollary 1. *Let $\vartheta\in\mathbb{C}$ and $-1\le B<A\le 1$. Suppose also that $\dfrac{1+Az}{1+Bz}$ satisfies the condition (6). If $f(z)\in\mathcal{A}$ satisfying the following subordination*

$$(1-\vartheta)\left(\frac{D_{\lambda}^{m}(\alpha,\beta)f(z)}{z}\right)^{\delta}+\vartheta\left(\frac{D_{\lambda}^{m+1}(\alpha,\beta)f(z)}{D_{\lambda}^{m}(\alpha,\beta)f(z)}\right)\left(\frac{D_{\lambda}^{m}(\alpha,\beta)f(z)}{z}\right)^{\delta} \prec\frac{1+Az}{1+Bz}+\frac{\lambda\vartheta(A-B)z}{\delta(1+Bz)^{2}},$$

then

$$\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta \prec \frac{1+Az}{1+Bz},$$

and $\frac{1+Az}{1+Bz}$ *is the best dominant.*

Theorem 3. *Let* $q(z)$ *be convex univalent in* $\mathbb{U}$, $\vartheta \in \mathbb{C}$, *with* $\mathcal{R}e(\vartheta) > 0$. *Also let* $\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta \in \mathcal{H}[q(0),1] \cap Q$ *and*

$$(1-\vartheta)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta + \vartheta\left(\frac{D_\lambda^{m+1}(\alpha,\beta)f(z)}{D_\lambda^m(\alpha,\beta)f(z)}\right)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta$$

be univalent in U. If

$$q(z) + \frac{\lambda\vartheta}{\delta} zq^{'}(z) \prec (1-\vartheta)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta + \vartheta\left(\frac{D_\lambda^{m+1}(\alpha,\beta)f(z)}{D_\lambda^m(\alpha,\beta)f(z)}\right)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta,$$

then

$$q(z) \prec \left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta,$$

and $q(z)$ *is the best subdominant.*

Proof. Let $p(z)$ be defined by (4). Then

$$q(z) + \frac{\lambda\vartheta}{\delta} zq^{'}(z) \prec (1-\vartheta)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta + \vartheta\left(\frac{D_\lambda^{m+1}(\alpha,\beta)f(z)}{D_\lambda^m(\alpha,\beta)f(z)}\right)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta = p(z) + \frac{\lambda\vartheta}{\delta} zp^{'}(z).$$

An application of Lemma 3 yields the assertion of Theorem 3. □

Corollary 2. *Let* $q(z)$ *be convex univalent in* $\mathbb{U}$ *and* $-1 \le B < A \le 1$,$\vartheta \in \mathbb{C}$ *with* $\mathcal{R}e(\vartheta) > 0$. *Also let* $\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta \in \mathcal{H}[q(0),1] \cap Q$ *and*

$$(1-\vartheta)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta + \vartheta\left(\frac{D_\lambda^{m+1}(\alpha,\beta)f(z)}{D_\lambda^m(\alpha,\beta)f(z)}\right)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta$$

be univalent in U. If

$$\frac{1+Az}{1+Bz} + \frac{\lambda\vartheta(A-B)z}{\delta(1+Bz)^2} \prec (1-\vartheta)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta + \vartheta\left(\frac{D_\lambda^{m+1}(\alpha,\beta)f(z)}{D_\lambda^m(\alpha,\beta)f(z)}\right)\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta,$$

then

$$\frac{1+Az}{1+Bz} \prec \left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta,$$

and $\frac{1+Az}{1+Bz}$ *is the best subdominant.*

Theorem 4. *Let $f(z) \in \mathcal{S}_{\vartheta}^{m,\delta}(\lambda,\alpha,\beta,A,B)$, then*

$$\inf_{z\in\mathbb{U}} \mathcal{R}e\left\{\frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{\delta}{\lambda n\vartheta}-1}du\right\} < \mathcal{R}e\left\{\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta\right\}$$
$$< \sup_{z\in\mathbb{U}} \mathcal{R}e\left\{\frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{\delta}{\lambda n\vartheta}-1}du\right\},$$

where $\vartheta \in \mathbb{C}, \mathcal{R}e(\delta) > 0, m \in \mathbb{N}_0, \lambda \geq 0, -1 \leq B \leq 1$ and $A \neq B \in \mathbb{N}_0$.

Proof. Suppose that $f(z) \in \mathcal{S}_{\vartheta}^{m,\delta}(\lambda,\alpha,\beta,A,B)$, then from Theorem 1 we know that

$$\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta \prec \frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{\delta}{\lambda n\vartheta}-1}du.$$

Thus, from the definition of the subordination, we get

$$\mathcal{R}e\left\{\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta\right\} > \inf_{z\in\mathbb{U}} \mathcal{R}e\left\{\frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{\delta}{\lambda n\vartheta}-1}du\right\}$$

and

$$\mathcal{R}e\left\{\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta\right\} < \sup_{z\in\mathbb{U}} \mathcal{R}e\left\{\frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{\delta}{\lambda n\vartheta}-1}du\right\}$$

□

Corollary 3. *Let $\vartheta \in \mathbb{C}, \mathcal{R}e(\delta) > 0, m \in \mathbb{N}_0, \lambda \geq 0$ and $-1 \leq B < A \leq 1$. If $f(z) \in \mathcal{S}_{\vartheta}^{m,\delta}(\lambda,\alpha,\beta,A,B)$, then*

$$\frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1-Au}{1-Bu}u^{\frac{\delta}{\lambda n\vartheta}-1}du < \mathcal{R}e\left\{\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta\right\}$$
$$< \frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Au}{1+Bu}u^{\frac{\delta}{\lambda n\vartheta}-1}du \quad (z \in \mathbb{U}).$$

Proof. Suppose that $f(z) \in \mathcal{S}_{\vartheta}^{m,\delta}(\lambda,\alpha,\beta,A,B)$, then from Theorem 1 we know that

$$\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta \prec \frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{\delta}{\lambda n\vartheta}-1}du \tag{9}$$

Thus, from the definition of the subordination, we get

$$\begin{aligned}\mathcal{R}e\left\{\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta\right\} &> \inf_{z\in\mathbb{U}} \mathcal{R}e\left\{\frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{\delta}{\lambda n\vartheta}-1}du\right\}\\ &\geq \frac{\delta}{\lambda n\vartheta}\int_0^1 \inf_{z\in\mathbb{U}}\left\{\frac{1+Azu}{1+Bzu}\right\}u^{\frac{\delta}{\lambda n\vartheta}-1}du\\ &> \frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1-Au}{1-Bu}u^{\frac{\delta}{\lambda n\vartheta}-1}du,\end{aligned}$$

and

$$\begin{aligned}\mathcal{R}e\left\{\left(\frac{D_\lambda^m(\alpha,\beta)f(z)}{z}\right)^\delta\right\} &< \sup_{z\in\mathbb{U}}\mathcal{R}e\left\{\frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Azu}{1+Bzu}u^{\frac{\delta}{\lambda n\vartheta}-1}du\right\}\\ &\le \frac{\delta}{\lambda n\vartheta}\int_0^1 \sup_{z\in\mathbb{U}}\left\{\frac{1+Azu}{1+Bzu}\right\}u^{\frac{\delta}{\lambda n\vartheta}-1}du\\ &< \frac{\delta}{\lambda n\vartheta}\int_0^1 \frac{1+Au}{1+Bu}u^{\frac{\delta}{\lambda n\vartheta}-1}du,\end{aligned}$$

which proves the result. □

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