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A SIMPLIFIED MATHEMATICAL MODEL OF THE UNIVERSE EXPANSION AND AN ESTIMATION OF ITS LIFE TIME DURATION

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Abstract. In this paper the authors represent their argument in establishing a most simple Mathematical Model as they could, in order to represent the Universe Expansion from the moment of its apparition (Big-Bang). We start from the fact that out of the total of 13.7 billion of years elapsed from this moment (B-B) up to the present, on the last portion of this interval about 90% of it, the curve which represent the Universe spatial dimension variation in function of time can be equivalent, with a sufficient large precision, to an arc of a circle.

1. Introduction

In a previous paper [1] the authors elaborated a Mathematical Model for the Universe Expansion. This model established the mathematical relation between "the dimension (the size) of the Universe" and the time elapsed from the moment of the Bib-Bang (B-B), on its road of continuous expansion. In order to develop an easier mathematical reasoning, in [1], we operated with the adimensional measures δ (for dimensions) and τ (for time). Thus, δ represents the relation between the Universe dimension at a certain given time (denoted by l) and its maximum actual dimension (denoted with l_{max}), admitted in the scientific world as being $l_{max} = 93 \times 10^9$ light-years (l-y). On the other side, τ represents the relation between the elapsed time, at a certain given moment, from the "explosion" (B-B), denoted with t and the total elapsed time up to the present admitted to be $t_{max} = 13.7 \times 10^9$ years.

In [1] we established a mathematical relation $\delta = f(\tau)$ which represents faithfully the curve which represents this function established by astronomic measurements and made final based on some elements from the domain of the Cosmology. This curve is given from measurements and is represented in Fig. 1. Practically, the graphical representation of the Mathematical Model elaborated in [1] is overlapping with the curve of Fig. 1 in the domain of $\tau = (0.05...1.0)$.

As a matter of fact, this domain presents a maximum interest, because in these limits the Universe is manifesting under its material forms also (stars, planets, galaxies), as we mentioned in [1]. Also, in Fig. 1, if we extend with a gentile variation the part of the curve which represents the function $\delta = f(\tau)$ in the domain $\tau = (0...0.1)$ then the interrupted line will meet the $O\delta$ axis in the point A, having the coordinates $\tau = 0$; $\delta_A = 0.59$. This fact will be useful in the section which follows. Despite of the fact that by applying this Mathematical Model presented in [1] brings us to very exact results, this contains a very large number of algebraic and trigonometric operations. Considering that, for the Cosmology phenomena, as it is also the case of the Universe Expansion, there is no need for an extreme precision in calculations, in this paper we intend to find a simpler, but sufficiently precise Mathematical Model for the Universe Expansion. In

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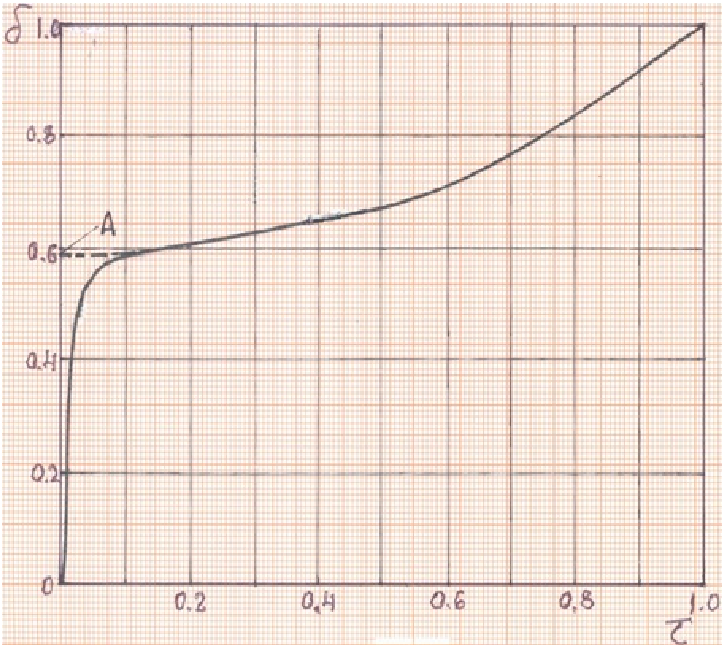

FIGURE 1.

what follows we will introduce this model. Also, the development of this model brought us to the possibility to estimate the life duration and the final size of the Universe in one of its present hypothesis, regarding its "disappearance".

2. The Symplified Mathematical Model of the Universe Expansion

Further on, to make easier our mathematical reasoning, we proceed changing the scale which represents τ in Fig. 1, thus such the value $\tau = 1.0$ to have the same dimension with $\delta = 1.0$. In this way in Fig. 2 appears the "square" of the coordinates δ and τ.

Transferring the values of the function $\delta = f(\tau)$ of Fig. 1 and considering the new representation scale for τ, we obtain the curve "a" in Fig. 2. We observe that this curve can be equivalent, with sufficient approximation, with an arc of a circle with the center situated on the extension of the $O\delta$ axis, at values above $\delta = 1.0$ and which passes through (or close to) the intersection point of $O\delta$ axis with the curve "a". The point A of Fig. 1 and Fig. 2 has the same coordinates. We have to consider the fact that the curve of the function $\delta = f(\tau)$ of Fig. 1 was extended by us (with an interrupted line) in the domain $\tau = (0...0.1)$ and therefore equivalence of this curve with the arc of circle represented by the curve "a" of Fig. 2 is valid only in the domain $\tau = (0.1...1.0)$, thus on about 90% of time t_{max}. It is known the following relation between the radius of the circle R, the length of the cord $AB = h$ and the arrow $CD = r$ as

$$R = r/2 + h^2/8r, \tag{1}$$

In our case, considering that in Fig. 2, $1cm$ represents 0.1τ and respectively $1cm$ represents 0.1δ, measuring we obtain $h = 10.8cm$ and $r = 1.1cm$. Introducing these values in relation (1) we obtain $R = 13.8cm$; expressed as an adimensional measure (same as with τ and δ) we have $R = 1.38$. Tracing the portion of the circle arc with this radius, the curve "b" which will superpose almost perfectly with the curve "a", intersects

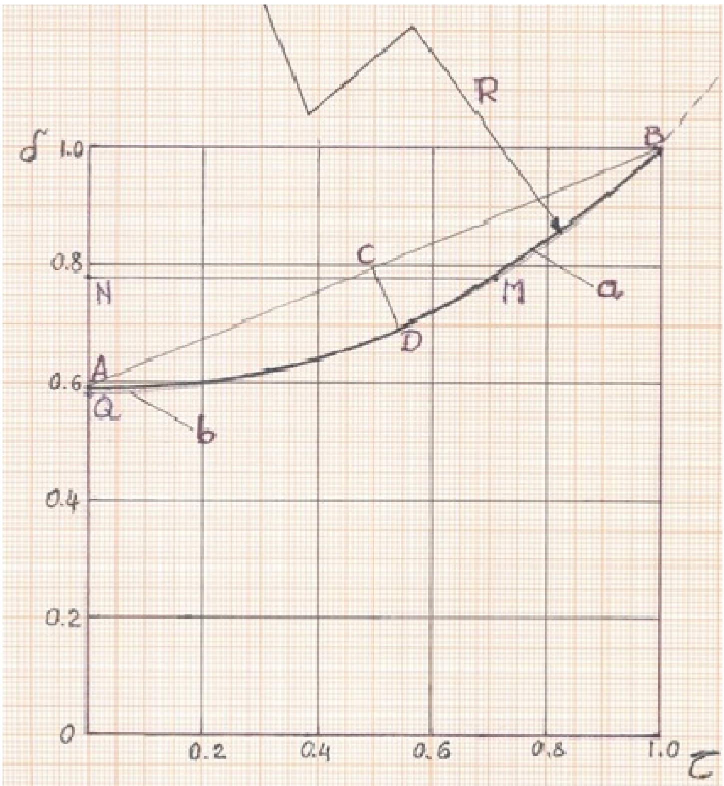

FIGURE 2.

the $O\delta$ axis in a point Q of coordinates $\tau = 0$ and $\delta = 0.58$ situated very close with the point A.

If we refer to a point M from this arc, on the curve "b" between $\tau_N = 0$ and τ_M, we observe that the right triangle having the vertexes M, N and the center of the circle of radius R (situated in the exterior of the figure frame), we have the relation (Pythagorean Theorem):

$$R^2 = \tau^2 + (R + \delta_0 - \delta)^2, \tag{2}$$

where $\tau = \tau_M$ and δ are the actual coordinates of the point M. The positive real root of δ from equation (2) constitutes the solution for the function $\delta = f(\tau)$ as such:

$$\delta = R + \delta_0 - (R^2 - \tau^2)^{1/2}. \tag{3}$$

Introducing in the relation (3) the values of the adimensional measures mentioned above, such as $R = 1.38$ and $\delta = 0.58$, we obtain

$$\delta = 1.96 - (1.9044 - \tau^2)^{1/2}. \tag{4}$$

For δ calculated in function of τ, in our case, we obtain values which differ from these contained in the curve "a", with deviations of maximum $+1.0\%$ and respectively -1.9%, thing which can be considered acceptable.

3. An Estimation of the Universe Life Duration, Based on Hypotheses of its Disappearance

In the previous section of this paper, we showed that the variation of τ between $\tau = 0$ and $\tau = 1.0$, represents the time laps between the moment of the Big-Bang (B-B) and the present time. In other words, in this domain of the values for τ, this represents the past of the Universe. Logically, the values for τ larger from 1 ($\tau > 1.0$) should represent the future of the Universe. If we admit that the evolution of δ in function of τ will develop in the same way as to the present, then the graphical representation of this fact could be considered having for the first successive period, from the extension for $\tau > 1.0$ of the circle b in Fig. 2 (traced with an interrupted line). We did not continue to trace this line for a longer period of time because of the limited frame in the figure. If we refer to the velocity of the Universe Expansion, evidently this is given by the value of the trigonometric tangent of the angle made by the geometric tangent to the circle "b" with $O\tau$ axis. It is known the fact that if we refer to the infinitesimal dimensions, this value represents the derivative of the function δ with respect of the variable τ. Taking the derivative of δ with respect to τ in equation (4) we obtain:

$$v = d\delta/d\tau = \tau(1.9044 - \tau^2)^{-1/2}. \tag{5}$$

The velocity v will have an infinite value when

$$(1.9044 - \tau^2)^{1/2} = 0. \tag{6}$$

Thus

$$1.9044 - \tau^2 = 0. \tag{7}$$

On the other side, one of the hypotheses regarding the Universe Disappearance or a beginning of a new cycle of Contraction-Expansion, based on the so called phenomena of a "Big Freeze" [2], brings us to a conclusion that: "All stars will die, all things will decompose and what will remain will be a mixture of particles and radiations. By that time, also the energy of this mixture will disappear under the effect of the Universe Expansion ...". Surely, we can't imagine that a such proportion of this expansion can be produced unless the velocity has an infinite value $v_f = \infty$. The index "f" from the velocity symbol comes from the word "final". Returning to the relation (7) we have in this case that the adimensional time of disappearance will be:

$$\tau_f = 1.9044^{1/2} = 1.38 \tag{8}$$

and the physical disappearance time will have the value $\tau_f = 1.38 \times 13.7 = 18.9$ billion years. Considering Fig. 2 this means that τ_f will have the value $\tau_f = R = 1.38$. Surely for this value of the coordinate τ the geometrical tangent to the circle "b" becomes parallel with the $O\delta$ axis and the value of the trigonometric tangent of the angle made by it with $O\tau$ axis becomes infinite. If we accept as valid the hypothesis of "Big Freeze" and using the disappearance time calculated above, because from the moment of the B-B up to the present passing 13.7 billion years, we still have that the life duration of the Universe could be of 18.9 - 13.7 = 5.2 billion years. Regarding the size of the Universe, again from Fig. 2 we have that corresponding with the time τ_f we will have that

$$1.9044 - \tau^2 = 0. \tag{9}$$

Thus $\delta_f = 0.58 + 1.38 = 1.96$. The physical size (measure) of the Universe, in this case, will be $1.96 \times 93 \times 10^9 = 182.3$ billion l-y. In this way, it results that starting the present time up to the disappearance, the Universe expansion will register an increase of $(182.3 - 93) \times 10^9 = 89.3$ billion l-y.

4. Conclusions

From the fact that in the domain of $\tau = (0...0.1)$, the curve which represents the Universe expansion (Fig. 1 and Fig. 2, curve "a") coincide, with a sufficient precision, with a circle arc (curve "b" of Fig. 2) we established a very simple relation (4) which represents the dependency of δ for τ. We remember that the adimensional measure τ is the relation between the time t (elapsed from the moment of B-B and the moment taken in consideration) and its maximum value (from B-B up to the present), $t_{max} = 13.7$ billion years. Similarly, the adimensional measure δ represents the relation between the spatial dimension of the Universe (l) at the moment τ and its dimension at the present time, $l_{max} = 93$ billion l-y. The deviations in the values of the measure δ calculated with the relation (4) compared with the values from the curve is based on the astronomic measures (Fig. 2, curve "a") varies in the limits 1.0% and -1.9%, thing which can be considered acceptable. Considering that the Universe destruction could follow conform to the hypothesis of the "Big Freeze" and that its expansion continues to develop again in concordance with the mentioned circle (extended for $\tau > 1.0$) it results that this "cataclysm" could have place in about 5.2 billion years. Corresponding to the time when this could happen, then the spatial dimension of the Universe will be of 182.3 billion l-y, thus it will "increase" starting this moment with another 89.3 billion l-y.

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