

ON THE CERTAIN SUBCLASSES OF UNIVALENT FUNCTIONS
ASSOCIATED WITH THE TSCHEBYSCHIEFF POLYNOMIALS

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ABSTRACT. In this work, considering certain subclasses of univalent functions and using the Tschebyscheff polynomials, we obtain coefficient expansions and solve the Fekete-Szegő problem for the functions that belong to these new subclasses.

1. INTRODUCTION

Denote by D the unit disc of the complex plane, $D = \{z \in \mathbb{C} : |z| < 1\}$, A the class of functions analytic in D , satisfying the conditions

$$f(0) = 0 \text{ and } f'(0) = 1.$$

Then each function f in A has the Taylor expansion

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \quad (1)$$

Also let S be the subclass of A consisting of functions the form (1) which are also univalent in D .

Further, we denote the class of starlike functions in A by S^* . It is well known that a function $f \in A$ is in S^* if and only if

$$\Re \left(\frac{z f'(z)}{f(z)} \right) > 0,$$

and denote by K the subclass of S of convex functions, so that $f \in K$ if, and only if, for $z \in D$

$$\Re \left(1 + \frac{z f''(z)}{f'(z)} \right) > 0.$$

If the functions f and g are analytic in D , then f is said to be subordinate to g , written as

$$f(z) \prec g(z), \quad (z \in D)$$

if there exists a Schwarz function $w(z)$, analytic in D , with

$$w(0) = 0 \text{ and } |w(z)| < 1 \quad (z \in D)$$

such that

$$f(z) = g(w(z)) \quad (z \in D).$$

Ma and Minda [6] unified various subclasses of starlike and convex functions for which either of the quantity $\frac{z f'(z)}{f(z)}$ or $1 + \frac{z f''(z)}{f'(z)}$ is subordinate to a more general superordinate function. For this purpose, they considered an analytic function φ with positive real part in D , $\varphi(0) = 0$, $\varphi'(0) > 0$, and φ maps D onto a region starlike with respect to 1

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and symmetric with respect to the real axis. The class of Ma-Minda starlike functions consists of functions $f \in A$ satisfying the subordination $\frac{zf'(z)}{f(z)} \prec \varphi(z)$. Similarly, the class of Ma-Minda convex functions consists of functions $f \in A$ satisfying the subordination $1 + \frac{zf''(z)}{f'(z)} \prec \varphi(z)$.

A classical theorem of Fekete-Szegő [3] states that for $f \in S$ of the form (1), the functional $|a_3 - \mu a_2^2|$ satisfies the inequality:

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu, & \mu \leq 0 \\ 1 + \exp\left(\frac{-2\mu}{1-\mu}\right), & 0 \leq \mu \leq 1 \\ 4\mu - 3, & \mu \geq 1 \end{cases} .$$

Keogh and Merkes [4], in 1969, obtained the sharp upper bound of the Fekete-Szegő functional $|a_3 - \mu a_2^2|$ for functions in some subclasses of S . The functional has since received great attention, particularly in many subclasses of the family of univalent functions (see for example, [1], [2], [8], [9]).

In the case of a real variable t on $(-1, 1)$, the expressions,

$$T_n(t) = \cos n\theta, \quad t = \cos \theta$$

uniquely define polynomials of the n th degree, the so called Tschebyscheff polynomials (see [7]).

The Tschebyscheff polynomials $T_n(t)$, $t \in (-1, 1)$ have the generating function of the form

$$\sum_{n=0}^{\infty} T_n(t) z^n = \frac{1 - tz}{1 - 2tz + z^2} \quad (z \in D).$$

Thus, we write

$$T(z, t) = 1 + L_1(t)z + L_2(t)z^2 + \dots \quad (z \in D).$$

Also it is known that

$$L_{n+1}(t) - 2tL_n(t) + L_{n-1}(t) = 0$$

and

$$\begin{aligned} L_0(t) &= 1 \\ L_1(t) &= t \\ L_2(t) &= 2t^2 - 1, \\ L_3(t) &= 4t^3 - 3t, \\ &\vdots \end{aligned} \tag{2}$$

The classes introduced in this paper are motivated by the corresponding classes investigated in [5]. We establish bounds for the coefficients, and solve the Fekete-Szegő problem for the functions that belong to these new classes. Furthermore, several related classes are also considered.

2. MAIN RESULTS

Next we consider the following new subclasses of A .

Definition 1. A function $f \in A$ is said to be in the class $M_\lambda(t)$, $0 \leq \lambda \leq 1$ and $t \in (\frac{1}{2}, 1]$, if the following subordination hold

$$\lambda f'(z) + (1 - \lambda) \frac{zf'(z)}{f(z)} \prec T(z, t) := \frac{1 - tz}{1 - 2tz + z^2} \quad (z \in D). \tag{3}$$

Note that

$$M_0(t) = \left\{ f \in A : \frac{zf'(z)}{f(z)} \prec T(z, t) \right\}$$

and

$$M_1(t) = \{f \in A : f'(z) \prec T(z, t)\}.$$

Theorem 1. Let the function $f(z)$ given by (1) be in the class $M_\lambda(t)$. Then

$$|a_2| \leq \frac{t}{1 + \lambda}$$

and

$$|a_3| \leq \frac{(2\lambda^2 + 3\lambda + 3)t^2}{(2 + \lambda)(1 + \lambda)^2} + \frac{t}{2 + \lambda} - \frac{1}{2 + \lambda}.$$

Proof. Let $f \in M_\lambda(t)$. From (3), we have

$$\lambda f'(z) + (1 - \lambda) \frac{zf'(z)}{f(z)} = 1 + L_1(t)w(z) + L_2(t)w^2(z) + \dots, \tag{4}$$

for some analytic functions w such that $w(0) = 0$ and $|w(z)| < 1$, for all $z \in D$. From the equalities (4), we obtain that

$$\lambda f'(z) + (1 - \lambda) \frac{zf'(z)}{f(z)} = 1 + L_1(t)c_1z + [L_1(t)c_2 + L_2(t)c_1^2]z^2 + \dots. \tag{5}$$

It is fairly well-known that if $|w(z)| = |c_1z + c_2z^2 + c_3z^3 + \dots| < 1$, $z \in D$, then

$$|c_j| \leq 1, \text{ for all } j \in \mathbb{N}; \tag{6}$$

and

$$|c_2 - \mu c_1^2| \leq \max\{1, |\mu|\}, \text{ for all } \mu \in \mathbb{R}. \tag{7}$$

It follows from (5) that

$$(1 + \lambda)a_2 = L_1(t)c_1, \tag{8}$$

$$(2 + \lambda)a_3 - (1 - \lambda)a_2^2 = L_1(t)c_2 + L_2(t)c_1^2. \tag{9}$$

From (2) and (8) we obtain

$$|a_2| \leq \frac{t}{1 + \lambda}. \tag{10}$$

Next, in order to find the bound on $|a_3|$, by using (8) in (9), we obtain

$$(2 + \lambda)a_3 = L_1(t)c_2 + \left\{ L_2(t) + \frac{(1 - \lambda)}{(1 + \lambda)^2} L_1^2(t) \right\} c_1^2. \tag{11}$$

Then, in view of (2) and (6), we have from (11)

$$|a_3| \leq \frac{(2\lambda^2 + 3\lambda + 3)t^2}{(2 + \lambda)(1 + \lambda)^2} + \frac{t}{2 + \lambda} - \frac{1}{2 + \lambda}.$$

□

Our next theorem gives the Fekete-Szegő inequality for functions in the class $M_\lambda(t)$.

Theorem 2. Let f given by (1) be in the class $M_\lambda(t)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{t}{2+\lambda}; & \text{for } \mu \in [\mu_1, \mu_2] \\ \frac{t}{2+\lambda} \left| \frac{2t^2-1}{t} + \frac{(1-\lambda)}{(1+\lambda)^2}t - \mu \frac{(2+\lambda)}{(1+\lambda)^2}t \right|; & \text{for } \mu \notin [\mu_1, \mu_2] \end{cases}$$

where

$$\mu_1 = \frac{(2\lambda^2+3\lambda+3)t^2-(1+\lambda)^2(1+t)}{(2+\lambda)t^2}, \quad \mu_2 = \frac{(2\lambda^2+3\lambda+3)t^2-(1+\lambda)^2(1-t)}{(2+\lambda)t^2}.$$

Proof. From (8) and (11)

$$|a_3 - \mu a_2^2| = \frac{L_1(t)}{2+\lambda} \left| c_2 + \left\{ \frac{L_2(t)}{L_1(t)} + \frac{1-\lambda}{(1+\lambda)^2}L_1(t) - \mu \frac{(2+\lambda)L_1(t)}{(1+\lambda)^2} \right\} c_1^2 \right|.$$

Then, in view of (7), we conclude that

$$|a_3 - \mu a_2^2| \leq \frac{L_1(t)}{2+\lambda} \max \left\{ 1, \left| \frac{L_2(t)}{L_1(t)} + \frac{1-\lambda}{(1+\lambda)^2}L_1(t) - \mu \frac{(2+\lambda)L_1(t)}{(1+\lambda)^2} \right| \right\}. \quad (12)$$

Finally, by using (2) in (12)

$$|a_3 - \mu a_2^2| \leq \frac{t}{1+2\lambda} \max \left\{ 1, \left| \frac{2t^2-1}{t} + \frac{(1-\lambda)}{(1+\lambda)^2}t - \mu \frac{(2+\lambda)}{(1+\lambda)^2}t \right| \right\}.$$

Because $t > 0$, we have

$$\begin{aligned} & \left| \frac{2t^2-1}{t} + \frac{(1-\lambda)}{(1+\lambda)^2}t - \mu \frac{(2+\lambda)}{(1+\lambda)^2}t \right| \leq 1 \\ & \Leftrightarrow \left\{ \frac{(2\lambda^2+3\lambda+3)t^2-(1+\lambda)^2(1+t)}{(2+\lambda)t^2} \leq \mu \leq \frac{(2\lambda^2+3\lambda+3)t^2-(1+\lambda)^2(1-t)}{(2+\lambda)t^2} \right\} \\ & \Leftrightarrow \mu_1 \leq \mu \leq \mu_2. \end{aligned}$$

□

Taking $\lambda = 1$ we get

Corollary 1. If $f \in M_1(t)$, then

$$\begin{aligned} |a_2| & \leq \frac{t}{3}; \\ |a_3| & \leq \frac{2t^2}{3} + \frac{t}{3} - \frac{1}{3}; \\ |a_3 - \mu a_2^2| & \leq \begin{cases} \frac{t}{3}; & \text{for } \mu \in [\mu_1, \mu_2] \\ \left| \frac{8t^2-4-3\mu t^2}{12} \right|; & \text{for } \mu \notin [\mu_1, \mu_2] \end{cases} \end{aligned}$$

Taking $\lambda = 0$ we get

Corollary 2. *If $f \in M_0(t)$, then*

$$|a_2| \leq t;$$

$$|a_3| \leq \frac{3t^2}{2} + \frac{t}{2} - \frac{1}{2};$$

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{t}{2}; & \text{for } \mu \in [\mu_1, \mu_2] \\ \left| \frac{3t^2 - 1 - 2\mu t^2}{2} \right|; & \text{for } \mu \notin [\mu_1, \mu_2] \end{cases}.$$

Definition 2. *A function $f \in A$ is said to be in the class $K(\lambda, t)$, $0 \leq \lambda \leq 1$ and $t \in (\frac{1}{2}, 1]$, if the following subordination hold*

$$(f'(z))^\lambda \left(\frac{zf'(z)}{f(z)} \right)^{(1-\lambda)} \prec T(z, t) := \frac{1-tz}{1-2tz+z^2} \quad (z \in D). \tag{13}$$

We see that $K(0, t) = M_0(t)$ and $K(1, t) = M_1(t)$.

Theorem 3. *Let the function $f(z)$ given by (1) be in the class $K(\lambda, t)$. Then*

$$|a_2| \leq \frac{t}{1+\lambda}$$

and

$$|a_3| \leq \frac{(3\lambda^2 + 7\lambda + 6)t^2}{2(2+\lambda)(1+\lambda)^2} + \frac{t}{2+\lambda} - \frac{1}{2+\lambda}.$$

Proof. Let $f \in K(\lambda, t)$. From (13), we obtain

$$(f'(z))^\lambda \left(\frac{zf'(z)}{f(z)} \right)^{(1-\lambda)} = 1 + L_1(t)c_1z + [L_1(t)c_2 + L_2(t)c_1^2]z^2 + \dots.$$

such that

$$(1+\lambda)a_2 = L_1(t)c_1, \tag{14}$$

$$(2+\lambda)a_3 - \frac{(2+\lambda)(1-\lambda)}{2}a_2^2 = L_1(t)c_2 + L_2(t)c_1^2. \tag{15}$$

From (2) and (14) we obtain

$$|a_2| \leq \frac{t}{1+\lambda}.$$

On the other hand, by using (14) in (15), we obtain

$$(2+\lambda)a_3 = L_1(t)c_2 + \left\{ L_2(t) + \frac{(2+\lambda)(1-\lambda)}{2(1+\lambda)^2}L_1^2(t) \right\} c_1^2. \tag{16}$$

Now using the facts that in (2) and taking the absolute values of either of the above equation in (16), we obtain

$$|a_3| \leq \frac{(3\lambda^2 + 7\lambda + 6)t^2}{2(2+\lambda)(1+\lambda)^2} + \frac{t}{2+\lambda} - \frac{1}{2+\lambda}.$$

□

Theorem 4. Let f given by (1) be in the class $K(\lambda, t)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{t}{2+\lambda}; & \text{for } \mu \in [\mu_1, \mu_2] \\ \frac{t}{2+\lambda} \left| \frac{2t^2-1}{t} + \frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2} t - \mu \frac{(2+\lambda)}{(1+\lambda)^2} t \right|; & \text{for } \mu \notin [\mu_1, \mu_2] \end{cases}.$$

where

$$\begin{aligned} \mu_1 &= \frac{(4(1+\lambda)^2(2+\lambda)+(1-\lambda)+3)t^2-2(1+\lambda)^2(2+\lambda)(1+t)}{2(2+\lambda)^2t^2}, \\ \mu_2 &= \frac{(4(1+\lambda)^2(2+\lambda)+(1-\lambda)+3)t^2-2(1+\lambda)^2(2+\lambda)(1-t)}{2(2+\lambda)^2t^2}. \end{aligned}$$

Proof. From (14) and (15)

$$|a_3 - \mu a_2^2| = \frac{L_1(t)}{2+\lambda} \left| c_2 + \left\{ \frac{L_2(t)}{L_1(t)} + \frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2} L_1(t) - \mu \frac{(2+\lambda)L_1(t)}{(1+\lambda)^2} \right\} c_1^2 \right|.$$

Then, in view of (7), we conclude that

$$|a_3 - \mu a_2^2| \leq \frac{U_1(t)}{2+\lambda} \max \left\{ 1, \left| \frac{L_2(t)}{L_1(t)} + \frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2} L_1(t) - \mu \frac{(2+\lambda)L_1(t)}{(1+\lambda)^2} \right| \right\}.$$

Finally, by using (2), we get

$$|a_3 - \mu a_2^2| \leq \frac{t}{2+\lambda} \max \left\{ 1, \left| \frac{2t^2-1}{t} + \frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2} t - \mu \frac{(2+\lambda)}{(1+\lambda)^2} t \right| \right\}.$$

Because $t > 0$, we have

$$\begin{aligned} \left| \frac{2t^2-1}{t} + \frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2} t - \mu \frac{(2+\lambda)}{(1+\lambda)^2} t \right| &\leq 1 \\ \Leftrightarrow \left\{ \frac{(3\lambda^2+7\lambda+6)t^2-2(1+\lambda)^2(1+t)}{2(2+\lambda)t^2} \leq \mu \leq \frac{(3\lambda^2+7\lambda+6)t^2-2(1+\lambda)^2(1-t)}{2(2+\lambda)t^2} \right\} \end{aligned}$$

□

Definition 3. A function $f \in A$ is said to be in the class $P(\lambda, t)$, $0 \leq \lambda \leq 1$ and $t \in (\frac{1}{2}, 1]$, if the following subordination hold

$$(f'(z))^\lambda \left(1 + \frac{zf''(z)}{f'(z)} \right)^{(1-\lambda)} \prec T(z, t) := \frac{1-tz}{1-2tz+z^2} \quad (z \in D).$$

Note that

$$P(0, t) = \left\{ f \in A : \left(1 + \frac{zf''(z)}{f'(z)} \right) \prec T(z, t) \right\}$$

and

$$P(1, t) = M_1(t).$$

Theorem 5. Let the function $f(z)$ given by (1) be in the class $P(\lambda, t)$. Then

$$|a_2| \leq \frac{t}{2}$$

and

$$|a_3| \leq \frac{(3-\lambda)t^2}{3(2-\lambda)} + \frac{t}{3(2-\lambda)} - \frac{1}{3(2-\lambda)}.$$

Proof. Let $f \in P(\lambda, t)$. From the last definition, we have

$$(f'(z))^\lambda \left(1 + \frac{zf''(z)}{f'(z)}\right)^{(1-\lambda)} = 1 + L_1(t)c_1z + [L_1(t)c_2 + L_2(t)c_1^2]z^2 + \dots \quad (17)$$

Comparing the corresponding coefficients of (17), yields

$$2a_2 = L_1(t)c_1, \quad (18)$$

$$3(2-\lambda)a_3 - 4(1-\lambda)a_2^2 = L_1(t)c_2 + L_2(t)c_1^2. \quad (19)$$

From (2) and (18) we obtain

$$|a_2| \leq \frac{t}{2}.$$

Next, in order to find the bound on $|a_3|$, by using (18) in (19), we obtain

$$3(2-\lambda)a_3 = L_1(t)c_2 + \{L_2(t) + (1-\lambda)L_1^2(t)\}c_1^2,$$

or, equivalently

$$|a_3| \leq \frac{(3-\lambda)t^2}{3(2-\lambda)} + \frac{t}{3(2-\lambda)} - \frac{1}{3(2-\lambda)}.$$

□

Theorem 6. Let f given by (1) be in the class $P(\lambda, t)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{t}{3(2-\lambda)}; & \text{for } \mu \in [\mu_1, \mu_2] \\ \frac{t}{3(2-\lambda)} \left| \frac{2t^2-1}{t} + (1-\lambda)t - \mu \frac{3(2-\lambda)}{4}t \right|; & \text{for } \mu \notin [\mu_1, \mu_2] \end{cases}.$$

where

$$\mu_1 = \frac{4(3-\lambda)t^2-4(1+t)}{3(2-\lambda)t^2}, \quad \mu_2 = \frac{4(3-\lambda)t^2-4(1-t)}{3(2-\lambda)t^2}.$$

Proof. From (18) and (19)

$$|a_3 - \mu a_2^2| = \frac{L_1(t)}{3(2-\lambda)} \left| c_2 + \left\{ \frac{L_2(t)}{L_1(t)} + (1-\lambda)L_1(t) - \mu \frac{3(2-\lambda)L_1(t)}{4} \right\} c_1^2 \right|.$$

Then, in view of (7), we conclude that

$$|a_3 - \mu a_2^2| \leq \frac{L_1(t)}{3(2-\lambda)} \max \left\{ 1, \left| \frac{L_2(t)}{L_1(t)} + (1-\lambda)L_1(t) - \mu \frac{3(2-\lambda)L_1(t)}{4} \right| \right\}. \quad (20)$$

Finally, by using (2) in (20)

$$|a_3 - \mu a_2^2| \leq \frac{t}{3(2-\lambda)} \max \left\{ 1, \left| \frac{2t^2-1}{t} + (1-\lambda)t - \mu \frac{3(2-\lambda)}{4}t \right| \right\}.$$

Because $t > 0$, we have

$$\begin{aligned} \left| \frac{2t^2-1}{t} + (1-\lambda)t - \mu \frac{3(2-\lambda)}{4}t \right| &\leq 1 \\ \Leftrightarrow \left\{ \frac{4(3-\lambda)t^2-4(1+t)}{3(2-\lambda)t^2} \leq \mu \leq \frac{4(3-\lambda)t^2-4(1-t)}{3(2-\lambda)t^2} \right\} \end{aligned}$$

□

Definition 4. A function $f \in A$ is said to be in the class $R(\lambda, t)$, $0 \leq \lambda \leq 1$ and $t \in (\frac{1}{2}, 1]$, if the following subordination hold

$$\left(\frac{f(z)}{z}\right)^\lambda \left(\frac{zf'(z)}{f(z)}\right)^{1-\lambda} \prec T(z, t) := \frac{1-tz}{1-2tz+z^2} \quad (z \in D).$$

We see that $R(0, t) = M_0(t)$.

Theorem 7. Let the function $f(z)$ given by (1) be in the class $R(\lambda, t)$. Then

$$|a_2| \leq t$$

and

$$|a_3| \leq \frac{(3-\lambda)t^2}{2-\lambda} + \frac{t}{2-\lambda} - \frac{1}{2-\lambda}.$$

Theorem 8. Let f given by (1) be in the class $R(\lambda, t)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{t}{2-\lambda}; & \text{for } \mu \in [\mu_1, \mu_2] \\ \frac{t}{2-\lambda} \left| \frac{2t^2-1}{t} + (1-\lambda)t - \mu(2-\lambda)t \right|; & \text{for } \mu \notin [\mu_1, \mu_2] \end{cases}.$$

where

$$\mu_1 = \frac{(3-\lambda)t^2 - (1+t)}{(2-\lambda)t^2}, \quad \mu_2 = \frac{(3-\lambda)t^2 - (1-t)}{(2-\lambda)t^2}.$$

Definition 5. A function $f \in A$ is said to be in the class $S(\lambda, t)$, $0 \leq \lambda \leq 1$ and $t \in (\frac{1}{2}, 1]$, if the following subordination hold

$$\left(\frac{f(z)}{z} \right)^\lambda \left(1 + \frac{zf''(z)}{f'(z)} \right)^{1-\lambda} \prec T(z, t) := \frac{1-tz}{1-2tz+z^2} \quad (z \in D).$$

We see that $S(0, t) = P(0, t)$.

Theorem 9. Let the function $f(z)$ given by (1) be in the class $S(\lambda, t)$. Then

$$|a_2| \leq \frac{t}{2-\lambda}$$

and

$$|a_3| \leq \frac{(3\lambda^2 - 23\lambda + 24)t^2}{2(2-\lambda)^2(6-5\lambda)} + \frac{t}{6-5\lambda} - \frac{1}{6-5\lambda}.$$

Theorem 10. Let f given by (1) be in the class $S(\lambda, t)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{t}{6-5\lambda}; & \text{for } \mu \in [\mu_1, \mu_2] \\ \frac{t}{6-5\lambda} \left| \frac{2t^2-1}{t} + \frac{(1-\lambda)(8+\lambda)}{2(2-\lambda)^2}t - \mu \frac{(6-5\lambda)}{(2-\lambda)^2}t \right|; & \text{for } \mu \notin [\mu_1, \mu_2] \end{cases}.$$

where

$$\mu_1 = \frac{(3\lambda^2 - 23\lambda + 24)t^2 - 2(2-\lambda)^2(1+t)}{2(6-5\lambda)t^2}, \quad \mu_2 = \frac{(3\lambda^2 - 23\lambda + 24)t^2 - 2(2-\lambda)^2(1-t)}{2(6-5\lambda)t^2}.$$

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