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ON THE CERTAIN SUBCLASSES OF UNIVALENT FUNCTIONS ASSOCIATED WITH THE TSCHEBYSCHEFF POLYNOMIALS

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Abstract. In this work, considering certain subclasses of univalent functions and using the Tschebyscheff polynomials, we obtain coefficient expansions and solve the Fekete-Szegö problem for the functions that belong to these new subclasses.

1. Introduction

Denote by D the unit disc of the complex plane, $D = \{z \in \mathbb{C} : |z| < 1\}$, A the class of functions analytic in D, satisfying the conditions

$$f(0) = 0 \text{ and } \quad f'(0) = 1.$$

Then each function f in A has the Taylor expansion

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1}$$

Also let S be the subclass of A consisting of functions the form (1) which are also univalent in D.

Further, we denote the class of starlike functions in A by S^*. It is well known that a function $f \in A$ is in S^* if and only if

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > 0,$$

and denote by K the subclass of S of convex functions, so that $f \in K$ if, and only if, for $z \in D$

$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > 0.$$

If the functions f and g are analytic in D, then f is said to be subordinate to g, written as

$$f(z) \prec g(z), \qquad (z \in D)$$

if there exists a Schwarz function $w(z)$, analytic in D, with

$$w(0) = 0 \text{ and } \quad |w(z)| < 1 \qquad (z \in D)$$

such that

$$f(z) = g(w(z)) \qquad (z \in D).$$

Ma and Minda [6] unified various subclasses of starlike and convex functions for which either of the quantity $\frac{zf'(z)}{f(z)}$ or $1 + \frac{zf''(z)}{f'(z)}$ is subordinate to a more general superordinate function. For this purpose, they considered an analytic function φ with positive real part in D, $\varphi(0) = 0$, $\varphi'(0) > 0$, and φ maps D onto a region starlike with respect to 1

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and symmetric with respect to the real axis. The class of Ma-Minda starlike functions consists of functions $f \in A$ satisfying the subordination $\frac{zf'(z)}{f(z)} \prec \varphi(z)$. Similarly, the class of Ma-Minda convex functions consists of functions $f \in A$ satisfying the subordination $1 + \frac{zf''(z)}{f'(z)} \prec \varphi(z)$.

A classical theorem of Fekete-Szegö [3] states that for $f \in S$ of the form (1), the functional $\left|a_3 - \mu a_2^2\right|$ satisfies the inequality:

$$\left|a_3 - \mu a_2^2\right| \leq \begin{cases} 3-4\mu, & \mu \leq 0 \\ 1+\exp\left(\frac{-2\mu}{1-\mu}\right), & 0 \leq \mu \leq 1 \\ 4\mu - 3, & \mu \geq 0 \end{cases}.$$

Keogh and Merkes [4], in 1969, obtained the sharp upper bound of the Fekete-Szegö functional $\left|a_3 - \mu a_2^2\right|$ for functions in some subclasses of S. The functional has since received great attention, particularly in many subclasses of the family of univalent functions (see for example, [1], [2], [8], [9]).

In the case of a real variable t on $(-1,1)$, the expressions,

$$T_n(t) = \cos n\theta, \quad t = \cos\theta$$

uniquely define polynomials of the nth degree, the so called Tschebyscheff polynomials (see [7]).

The Tschebyscheff polynomials $T_n(t),\ t \in (-1,1)$ have the generating function of the form

$$\sum_{n=0}^{\infty} T_n(t) z^n = \frac{1-tz}{1-2tz+z^2} \quad (z \in D).$$

Thus, we write

$$T(z,t) = 1 + L_1(t)z + L_2(t)z^2 + \cdots \quad (z \in D).$$

Also it is known that

$$L_{n+1}(t) - 2tL_n(t) + L_{n-1}(t) = 0$$

and

$$\begin{aligned} L_0(t) &= 1 \\ L_1(t) &= t \\ L_2(t) &= 2t^2 - 1, \\ L_3(t) &= 4t^3 - 3t, \\ &\vdots \end{aligned} \tag{2}$$

The classes introduced in this paper are motivated by the corresponding classes investigated in [5]. We establish bounds for the coefficients, and solve the Fekete-Szegö problem for the functions that belong to these new classes. Furthermore, several related classes are also considered.

2. Main results

Next we consider the following new subclasses of A.

Definition 1. *A function $f \in A$ is said to be in the class $M_\lambda(t)$, $0 \leq \lambda \leq 1$ and $t \in \left(\frac{1}{2}, 1\right]$, if the following subordination hold*

$$\lambda f'(z) + (1-\lambda)\frac{zf'(z)}{f(z)} \prec T(z,t) := \frac{1-tz}{1-2tz+z^2} \qquad (z \in D). \tag{3}$$

Note that

$$M_0(t) = \left\{ f \in A : \frac{zf'(z)}{f(z)} \prec T(z,t) \right\}$$

and

$$M_1(t) = \left\{ f \in A : f'(z) \prec T(z,t) \right\}.$$

Theorem 1. *Let the function $f(z)$ given by (1) be in the class $M_\lambda(t)$. Then*

$$|a_2| \leq \frac{t}{1+\lambda}$$

and

$$|a_3| \leq \frac{\left(2\lambda^2+3\lambda+3\right)t^2}{(2+\lambda)(1+\lambda)^2} + \frac{t}{2+\lambda} - \frac{1}{2+\lambda}.$$

Proof. Let $f \in M_\lambda(t)$. From (3), we have

$$\lambda f'(z) + (1-\lambda)\frac{zf'(z)}{f(z)} = 1 + L_1(t)w(z) + L_2(t)w^2(z) + \cdots, \tag{4}$$

for some analytic functions w such that $w(0) = 0$ and $|w(z)| < 1$, for all $z \in D$. From the equalities (4), we obtain that

$$\lambda f'(z) + (1-\lambda)\frac{zf'(z)}{f(z)} = 1 + L_1(t)c_1 z + \left[L_1(t)c_2 + L_2(t)c_1^2\right]z^2 + \cdots. \tag{5}$$

It is fairly well-known that if $|w(z)| = \left|c_1 z + c_2 z^2 + c_3 z^3 + \cdots\right| < 1$, $z \in D$, then

$$|c_j| \leq 1, \quad for \quad all \quad j \in \mathbb{N}; \tag{6}$$

and

$$\left|c_2 - \mu c_1^2\right| \leq \max\{1, |\mu|\}, \quad for \quad all \quad for\ \mu \in \mathbb{R}. \tag{7}$$

It follows from (5) that

$$(1+\lambda)a_2 = L_1(t)c_1, \tag{8}$$

$$(2+\lambda)a_3 - (1-\lambda)a_2^2 = L_1(t)c_2 + L_2(t)c_1^2. \tag{9}$$

From (2) and (8) we obtain

$$|a_2| \leq \frac{t}{1+\lambda}. \tag{10}$$

Next, in order to find the bound on $|a_3|$, by using (8) in (9), we obtain

$$(2+\lambda)a_3 = L_1(t)c_2 + \left\{ L_2(t) + \frac{(1-\lambda)}{(1+\lambda)^2}L_1^2(t) \right\} c_1^2. \tag{11}$$

Then, in view of (2) and (6) , we have from (11)

$$|a_3| \leq \frac{\left(2\lambda^2+3\lambda+3\right)t^2}{(2+\lambda)(1+\lambda)^2} + \frac{t}{2+\lambda} - \frac{1}{2+\lambda}.$$

□

Our next theorem gives the Fekete-Szegö inequality for functions in the class $M_\lambda(t)$.

Theorem 2. *Let f given by (1) be in the class $M_{\lambda}(t)$. Then*

$$\left|a_3-\mu a_2^2\right| \leq \begin{cases} \dfrac{t}{2+\lambda}; & for\ \mu \in [\mu_1,\mu_2] \\ \dfrac{t}{2+\lambda}\left|\frac{2t^2-1}{t}+\frac{(1-\lambda)}{(1+\lambda)^2}t-\mu\frac{(2+\lambda)}{(1+\lambda)^2}t\right|; & for\ \mu \notin [\mu_1,\mu_2] \end{cases}.$$

where

$$\mu_1=\frac{\left(2\lambda^2+3\lambda+3\right)t^2-(1+\lambda)^2(1+t)}{(2+\lambda)t^2}, \quad \mu_2=\frac{\left(2\lambda^2+3\lambda+3\right)t^2-(1+\lambda)^2(1-t)}{(2+\lambda)t^2}.$$

Proof. From (8) and (11)

$$\left|a_3-\mu a_2^2\right|=\frac{L_1(t)}{2+\lambda}\left|c_2+\left\{\frac{L_2(t)}{L_1(t)}+\frac{1-\lambda}{(1+\lambda)^2}L_1(t)-\mu\frac{(2+\lambda)\,L_1(t)}{(1+\lambda)^2}\right\}c_1^2\right|.$$

Then, in view of (7), we conclude that

$$\left|a_3-\mu a_2^2\right| \leq \frac{L_1(t)}{2+\lambda}\max\left\{1,\left|\frac{L_2(t)}{L_1(t)}+\frac{1-\lambda}{(1+\lambda)^2}L_1(t)-\mu\frac{(2+\lambda)\,L_1(t)}{(1+\lambda)^2}\right|\right\}. \tag{12}$$

Finally, by using (2) in (12)

$$\left|a_3-\mu a_2^2\right| \leq \frac{t}{1+2\lambda}\max\left\{1,\left|\frac{2t^2-1}{t}+\frac{(1-\lambda)}{(1+\lambda)^2}t-\mu\frac{(2+\lambda)}{(1+\lambda)^2}t\right|\right\}.$$

Because $t>0$, we have

$$\begin{aligned} &\left|\tfrac{2t^2-1}{t}+\tfrac{(1-\lambda)}{(1+\lambda)^2}t-\mu\tfrac{(2+\lambda)}{(1+\lambda)^2}t\right| \leq 1 \\ &\Leftrightarrow \left\{\tfrac{\left(2\lambda^2+3\lambda+3\right)t^2-(1+\lambda)^2(1+t)}{(2+\lambda)t^2} \leq \mu \leq \tfrac{\left(2\lambda^2+3\lambda+3\right)t^2-(1+\lambda)^2(1-t)}{(2+\lambda)t^2}\right\} \\ &\Leftrightarrow \mu_1 \leq \mu \leq \mu_2. \end{aligned}$$

□

Taking $\lambda=1$ we get

Corollary 1. *If $f \in M_1(t)$, then*

$$\begin{aligned} &|a_2| \leq \tfrac{t}{3}; \\ &|a_3| \leq \tfrac{2t^2}{3}+\tfrac{t}{3}-\tfrac{1}{3}; \\ &\left|a_3-\mu a_2^2\right| \leq \begin{cases} \dfrac{t}{3}; & for\ \mu \in [\mu_1,\mu_2] \\ \left|\frac{8t^2-4-3\mu t^2}{12}\right|; & for\ \mu \notin [\mu_1,\mu_2] \end{cases}. \end{aligned}$$

Taking $\lambda=0$ we get

Corollary 2. *If* $f \in M_0(t)$, *then*

$$|a_2| \leq t;$$

$$|a_3| \leq \tfrac{3t^2}{2} + \tfrac{t}{2} - \tfrac{1}{2};$$

$$\left|a_3 - \mu a_2^2\right| \leq \begin{cases} \dfrac{t}{2}; & \text{for } \mu \in [\mu_1, \mu_2] \\ \left|\frac{3t^2-1-2\mu t^2}{2}\right|; & \text{for } \mu \notin [\mu_1, \mu_2] \end{cases}.$$

Definition 2. *A function* $f \in A$ *is said to be in the class* $K(\lambda, t)$, $0 \leq \lambda \leq 1$ *and* $t \in \left(\frac{1}{2}, 1\right]$, *if the following subordination hold*

$$(f'(z))^{\lambda} \left(\frac{zf'(z)}{f(z)}\right)^{(1-\lambda)} \prec T(z,t) := \frac{1-tz}{1-2tz+z^2} \qquad (z \in D). \tag{13}$$

We see that $K(0,t) = M_0(t)$ and $K(1,t) = M_1(t)$.

Theorem 3. *Let the function* $f(z)$ *given by (1) be in the class* $K(\lambda, t)$. *Then*

$$|a_2| \leq \frac{t}{1+\lambda}$$

and

$$|a_3| \leq \frac{\left(3\lambda^2 + 7\lambda + 6\right)t^2}{2(2+\lambda)(1+\lambda)^2} + \frac{t}{2+\lambda} - \frac{1}{2+\lambda}.$$

Proof. Let $f \in K(\lambda, t)$. From (13), we obtain

$$(f'(z))^{\lambda} \left(\frac{zf'(z)}{f(z)}\right)^{(1-\lambda)} = 1 + L_1(t)c_1 z + \left[L_1(t)c_2 + L_2(t)c_1^2\right] z^2 + \cdots.$$

such that

$$(1+\lambda)\, a_2 = L_1(t)c_1, \tag{14}$$

$$(2+\lambda)\, a_3 - \frac{(2+\lambda)(1-\lambda)}{2} a_2^2 = L_1(t)c_2 + L_2(t)c_1^2. \tag{15}$$

From (2) and (14) we obtain

$$|a_2| \leq \frac{t}{1+\lambda}.$$

On the other hand, by using (14) in (15), we obtain

$$(2+\lambda)\, a_3 = L_1(t)c_2 + \left\{L_2(t) + \frac{(2+\lambda)(1-\lambda)}{2(1+\lambda)^2} L_1^2(t)\right\} c_1^2. \tag{16}$$

Now using the facts that in (2) and taking the absolute values of either of the above equation in (16), we obtain

$$|a_3| \leq \frac{\left(3\lambda^2 + 7\lambda + 6\right)t^2}{2(2+\lambda)(1+\lambda)^2} + \frac{t}{2+\lambda} - \frac{1}{2+\lambda}.$$

□

Theorem 4. *Let f given by (1) be in the class $K(\lambda, t)$. Then*

$$\left|a_3-\mu a_2^2\right| \leq \begin{cases} \dfrac{t}{2+\lambda}; & for\ \mu \in [\mu_1, \mu_2] \\ \dfrac{t}{2+\lambda}\left|\frac{2t^2-1}{t}+\frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2}t-\mu\frac{(2+\lambda)}{(1+\lambda)^2}t\right|; & for\ \mu \notin [\mu_1, \mu_2] \end{cases}.$$

where

$$\begin{aligned} \mu_1 &= \frac{\left(4(1+\lambda)^2(2+\lambda)+(1-\lambda)+3\right)t^2-2(1+\lambda)^2(2+\lambda)(1+t)}{2(2+\lambda)^2t^2}, \\ \mu_2 &= \frac{\left(4(1+\lambda)^2(2+\lambda)+(1-\lambda)+3\right)t^2-2(1+\lambda)^2(2+\lambda)(1-t)}{2(2+\lambda)^2t^2}. \end{aligned}$$

Proof. From (14) and (15)

$$\left|a_3-\mu a_2^2\right| = \frac{L_1(t)}{2+\lambda}\left|c_2+\left\{\frac{L_2(t)}{L_1(t)}+\frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2}L_1(t)-\mu\frac{(2+\lambda)L_1(t)}{(1+\lambda)^2}\right\}c_1^2\right|.$$

Then, in view of (7), we conclude that

$$\left|a_3-\mu a_2^2\right| \leq \frac{U_1(t)}{2+\lambda}\max\left\{1, \left|\frac{L_2(t)}{L_1(t)}+\frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2}L_1(t)-\mu\frac{(2+\lambda)L_1(t)}{(1+\lambda)^2}\right|\right\}.$$

Finally, by using (2), we get

$$\left|a_3-\mu a_2^2\right| \leq \frac{t}{2+\lambda}\max\left\{1, \left|\frac{2t^2-1}{t}+\frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2}t-\mu\frac{(2+\lambda)}{(1+\lambda)^2}t\right|\right\}.$$

Because $t>0$, we have

$$\begin{aligned} &\left|\frac{2t^2-1}{t}+\frac{(1-\lambda)(2+\lambda)}{2(1+\lambda)^2}t-\mu\frac{(2+\lambda)}{(1+\lambda)^2}t\right| \leq 1 \\ &\Leftrightarrow \left\{\frac{(3\lambda^2+7\lambda+6)t^2-2(1+\lambda)^2(1+t)}{2(2+\lambda)t^2} \leq \mu \leq \frac{(3\lambda^2+7\lambda+6)t^2-2(1+\lambda)^2(1-t)}{2(2+\lambda)t^2}\right\} \end{aligned}$$

□

Definition 3. *A function $f \in A$ is said to be in the class $P(\lambda, t)$, $0 \leq \lambda \leq 1$ and $t \in \left(\frac{1}{2}, 1\right]$, if the following subordination hold*

$$\left(f'(z)\right)^{\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right)^{(1-\lambda)} \prec T(z,t) := \frac{1-tz}{1-2tz+z^2} \qquad (z \in D).$$

Note that

$$P(0,t) = \left\{f \in A : \left(1+\frac{zf''(z)}{f'(z)}\right) \prec T(z,t)\right\}$$

and

$$P(1,t) = M_1(t).$$

Theorem 5. *Let the function $f(z)$ given by (1) be in the class $P(\lambda, t)$. Then*

$$|a_2| \leq \frac{t}{2}$$

and

$$|a_3| \leq \frac{(3-\lambda)t^2}{3(2-\lambda)}+\frac{t}{3(2-\lambda)}-\frac{1}{3(2-\lambda)}.$$

Proof. Let $f \in P(\lambda, t)$. From the last definition, we have

$$\left(f'(z)\right)^{\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right)^{(1-\lambda)} = 1 + L_1(t)c_1 z + \left[L_1(t)c_2 + L_2(t)c_1^2\right]z^2 + \cdots. \tag{17}$$

Comparing the corresponding coefficients of (17), yields

$$2a_2 = L_1(t)c_1, \tag{18}$$

$$3(2-\lambda)a_3 - 4(1-\lambda)a_2^2 = L_1(t)c_2 + L_2(t)c_1^2. \tag{19}$$

From (2) and (18) we obtain

$$|a_2| \le \frac{t}{2}.$$

Next, in order to find the bound on $|a_3|$, by using (18) in (19), we obtain

$$3(2-\lambda)a_3 = L_1(t)c_2 + \left\{L_2(t) + (1-\lambda)L_1^2(t)\right\}c_1^2,$$

or, equivalently

$$|a_3| \le \frac{(3-\lambda)t^2}{3(2-\lambda)} + \frac{t}{3(2-\lambda)} - \frac{1}{3(2-\lambda)}.$$

□

Theorem 6. *Let f given by (1) be in the class $P(\lambda, t)$. Then*

$$\left|a_3 - \mu a_2^2\right| \le \begin{cases} \dfrac{t}{3(2-\lambda)}; & for\ \mu \in [\mu_1, \mu_2] \\ \\ \dfrac{t}{3(2-\lambda)}\left|\frac{2t^2-1}{t} + (1-\lambda)t - \mu\frac{3(2-\lambda)}{4}t\right|; & for\ \mu \notin [\mu_1, \mu_2] \end{cases}.$$

where

$$\mu_1 = \tfrac{4(3-\lambda)t^2 - 4(1+t)}{3(2-\lambda)t^2}, \quad \mu_2 = \tfrac{4(3-\lambda)t^2 - 4(1-t)}{3(2-\lambda)t^2}.$$

Proof. From (18) and (19)

$$\left|a_3 - \mu a_2^2\right| = \frac{L_1(t)}{3(2-\lambda)}\left|c_2 + \left\{\frac{L_2(t)}{L_1(t)} + (1-\lambda)L_1(t) - \mu\frac{3(2-\lambda)L_1(t)}{4}\right\}c_1^2\right|.$$

Then, in view of (7), we conclude that

$$\left|a_3 - \mu a_2^2\right| \le \frac{L_1(t)}{3(2-\lambda)}\max\left\{1, \left|\frac{L_2(t)}{L_1(t)} + (1-\lambda)L_1(t) - \mu\frac{3(2-\lambda)L_1(t)}{4}\right|\right\}. \tag{20}$$

Finally, by using (2) in (20)

$$\left|a_3 - \mu a_2^2\right| \le \frac{t}{3(2-\lambda)}\max\left\{1, \left|\frac{2t^2-1}{t} + (1-\lambda)t - \mu\frac{3(2-\lambda)}{4}t\right|\right\}.$$

Because $t > 0$, we have

$$\left|\tfrac{2t^2-1}{t} + (1-\lambda)t - \mu\tfrac{3(2-\lambda)}{4}t\right| \le 1$$

$$\Leftrightarrow \left\{\tfrac{4(3-\lambda)t^2 - 4(1+t)}{3(2-\lambda)t^2} \le \mu \le \tfrac{4(3-\lambda)t^2 - 4(1-t)}{3(2-\lambda)t^2}\right\}$$

□

Definition 4. *A function $f \in A$ is said to be in the class $R(\lambda, t)$, $0 \le \lambda \le 1$ and $t \in \left(\frac{1}{2}, 1\right]$, if the following subordination hold*

$$\left(\frac{f(z)}{z}\right)^{\lambda}\left(\frac{zf'(z)}{f(z)}\right)^{1-\lambda} \prec T(z,t) := \frac{1-tz}{1-2tz+z^2} \qquad (z \in D).$$

We see that $R(0,t) = M_0(t)$.

Theorem 7. *Let the function $f(z)$ given by (1) be in the class $R(\lambda,t)$. Then*

$$|a_2| \leq t$$

and

$$|a_3| \leq \frac{(3-\lambda)t^2}{2-\lambda} + \frac{t}{2-\lambda} - \frac{1}{2-\lambda}.$$

Theorem 8. *Let f given by (1) be in the class $R(\lambda,t)$. Then*

$$\left|a_3 - \mu a_2^2\right| \leq \begin{cases} \dfrac{t}{2-\lambda}; & for\ \mu \in [\mu_1,\mu_2] \\ \dfrac{t}{2-\lambda}\left|\frac{2t^2-1}{t} + (1-\lambda)t - \mu(2-\lambda)t\right|; & for\ \mu \notin [\mu_1,\mu_2] \end{cases}.$$

where

$$\mu_1 = \tfrac{(3-\lambda)t^2-(1+t)}{(2-\lambda)t^2}, \quad \mu_2 = \tfrac{(3-\lambda)t^2-(1-t)}{(2-\lambda)t^2}.$$

Definition 5. *A function $f \in A$ is said to be in the class $S(\lambda,t)$, $0 \leq \lambda \leq 1$ and $t \in \left(\frac{1}{2},1\right]$, if the following subordination hold*

$$\left(\frac{f(z)}{z}\right)^{\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\lambda} \prec T(z,t) := \frac{1-tz}{1-2tz+z^2} \qquad (z \in D).$$

We see that $S(0,t) = P(0,t)$.

Theorem 9. *Let the function $f(z)$ given by (1) be in the class $S(\lambda,t)$. Then*

$$|a_2| \leq \frac{t}{2-\lambda}$$

and

$$|a_3| \leq \frac{\left(3\lambda^2-23\lambda+24\right)t^2}{2(2-\lambda)^2(6-5\lambda)} + \frac{t}{6-5\lambda} - \frac{1}{6-5\lambda}.$$

Theorem 10. *Let f given by (1) be in the class $S(\lambda,t)$. Then*

$$\left|a_3 - \mu a_2^2\right| \leq \begin{cases} \dfrac{t}{6-5\lambda}; & for\ \mu \in [\mu_1,\mu_2] \\ \dfrac{t}{6-5\lambda}\left|\frac{2t^2-1}{t} + \frac{(1-\lambda)(8+\lambda)}{2(2-\lambda)^2}t - \mu\frac{(6-5\lambda)}{(2-\lambda)^2}t\right|; & for\ \mu \notin [\mu_1,\mu_2] \end{cases}.$$

where

$$\mu_1 = \tfrac{\left(3\lambda^2-23\lambda+24\right)t^2-2(2-\lambda)^2(1+t)}{2(6-5\lambda)t^2}, \quad \mu_2 = \tfrac{\left(3\lambda^2-23\lambda+24\right)t^2-2(2-\lambda)^2(1-t)}{2(6-5\lambda)t^2}.$$

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