TJMM
8 (2016), No. 1, 15-20

A MATHEMATICAL MODEL FOR THE UNIVERSE EXPANSION

MALVINA BAICA AND MIRCEA CARDU

Abstract. In this paper we briefly introduce the Universe Expansion Phenomena and we comment about its graphical representation, in the spatial perspective, which appears in the specialized literature. Starting from this spatial representation, we trace the in plane representation of this phenomena. In order to find the mathematical model corresponding to the graphical representation of the Universe expansion we apply the paratrigonometric functions presented in [6]. Finally, we establish the needed mathematical model.

1. Introduction

In the scientific world is about unanimously recognized that the Universe which we actually see appeared during an explosion having an unimaginable force, known under the name of Big-Bang (B-B). From that "point" it is considered that both the space (accepted astronomically) and the time must be measured as fundamental elements in Physics. On the other side, as a result of ample astronomic observations and calculations, it is admissible the fact that this Universe was and is continuously in an expansion process (see [1], [2], [3], [4]). In the beginning this expansion was extremely accelerated (inflation) and after that it was produced with a reduced acceleration and in the period of the last (5-6) billions of years (from a total of about 13.7 billions of years after B-B) is also characterized by a higher value of the corresponding acceleration.

In order to give a more suggestive image of the Universe expansion, we created a representation in a spatial perspective of the increasing in time of the distance "l" which represents its "spatial measure" starting from the value 0 (zero), at the B-B moment up to the value of 93 billons light-years at the present time. Expressed in other dimensions used in the Cosmology, this value is equivalent with 2.85×10^4 MPc (by "Pc" we mean "Parsec"), respectively with 8.8×10^{23} km. This representation has the form of a revolving body having the time axis as the central axis, [5]. We named the respective cylindrical body as the "Expansion Bell" of the Universe (UEB).

In order to reach our purpose in this paper, we imagined a section of the (UEB) by a longitudinal plan, which comprise the time axis. Thus we have the graphical representation from Fig. 1. In Fig. 1 the indications regarding the diverse periods of the Universe expansions were taken from [5]. In this figure on the abscissa axis is represented the time, in divisions which are tenths from the total time of 13.7 billion years. Corresponding to the axis of the ordinates, the measure of "spatial dimensions" of the Universe is represented by the distance between the two symmetric branches (against the axis X-Y) of the (UEB). This dimension, as we have said, varies between 0 (zero) at the moment of the B-B and 93 billion of light-years (l-y).

Coming back to the (UEB) representations in the spatial perspective, as in [5] or in plan on the coordinates space-time as in Fig. 1, we remark that in neither case we should

2010 *Mathematics Subject Classification.* 03H05, 78A40, 81P10.
Key words and phrases. Paratrigonometry, Paratrigonometric functions, Universe, Universe expansion.

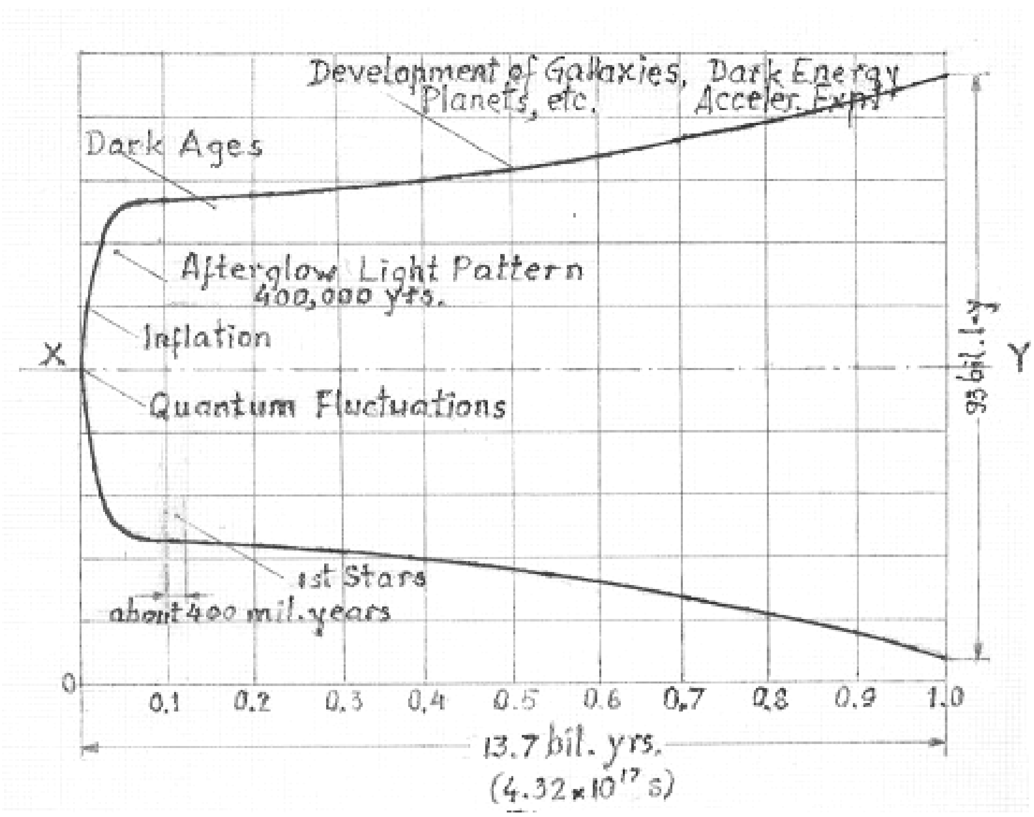

FIGURE 1.

not imagine that the Universe is comprised in the interior of this "bell" only and in its exterior having an absolute void. These representations were devised with the reason to find and for "time" a geometrical illustration, also. For this reason one of the three components of the space was sacrificed in the favor of the time. From the physical point of view however, we consider more suggestively the spatial virtual image promoted by the TV channel "Discovery" in which the Universe expansion in space is presented like a "ball of foam" whose dimension increases more and more as the time passes. In these conditions we must imagine the time as a measure which constantly and uniformly evolves in all directions of the space starting from the origin which is the moment of the *B-B*. In the next section we will present the argument which we applied in order to establish the proposed mathematical model.

2. The development of the mathematical model regarding the Universe expansion

In order to simplify the matter from the mathematical point of view, with reference to the data followed from Fig. 1 regarding the dimension l and the time t, we introduce the no dimensional measures δ and τ. Thus, δ represents the relation between the dimension l of the Universe at the moment t and its maximal actual dimension, $l_{max} = 93 \times 10^9$ *l-y* (light-years). In its turn, τ represents the relation between the current t, measured in years and the time passed from the moment of *B-B* up to the present. The Scholars

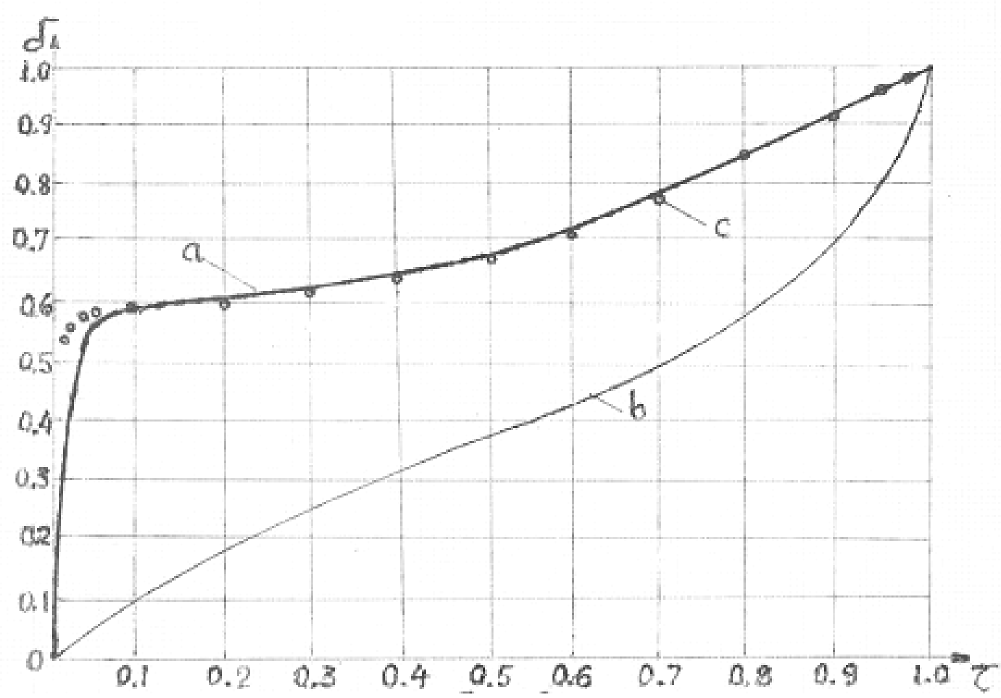

FIGURE 2.

who are working in the domain of the Cosmology, unanimously agreed with the value for $t_{max} = 13.7 \times 10^9$ years.

Having in our view the information from above, with the data from Fig. 1 we constructed the curve "a" of Fig. 2 which represents the function $\delta = f(\tau)$.

On the abscissa axis τ has values between $\tau_{min} = 0$ (the moment when B-B was produced) and $\tau_{max} = 1, 0$ ("present" moment).

On the ordinate axis δ has also, values between $\delta_{min} = 0$ (at the moment when B-B was produced) and $\delta_{max} = 1, 0$ (the "present" dimension of the Universe).

Evidently, the curve "a" of Fig. 2 has a complete unusual form compared with the curves of the functions generally seen in the Classical Mathematics. This is the reason why in our attempt to establish a Mathematical Model to find the relation between δ and τ, we appeal to the chapter 11 of an unconventional Mathematics which we developed in [6]. We figured out that the function "Sinus para-trigonometric of order k", that is $spr_k\delta$ in the domain of variation of angle $\delta = 0°...90°$ have its graphic representation close enough with the curve "a". The function $spr_k\delta$ is given by the relation:

$$spr_k\alpha = [1 + (tg\alpha)^k]^{-1/k} tg\alpha \ , \tag{1}$$

where $tg\alpha$ is the function "tangent" of the Classical Trigonometry.

In order to readapt to the coordinate system $\delta - \tau$ we replace α with $\omega = (90\tau)[°]$ and relation (1) becomes:

$$spr_k\omega = [1 + (tg\omega)^k]^{-1/k} tg\omega \ . \tag{2}$$

Exploring we arrived to the conclusion that the most adequate value for our case is $k = 0.7$. The curve "b" of Fig. 2 represents the function $spr_{0.7}\omega$. For simplification we replace $spr_k\omega$ with the function θ and considering that we choose $k = 0.7$ then the

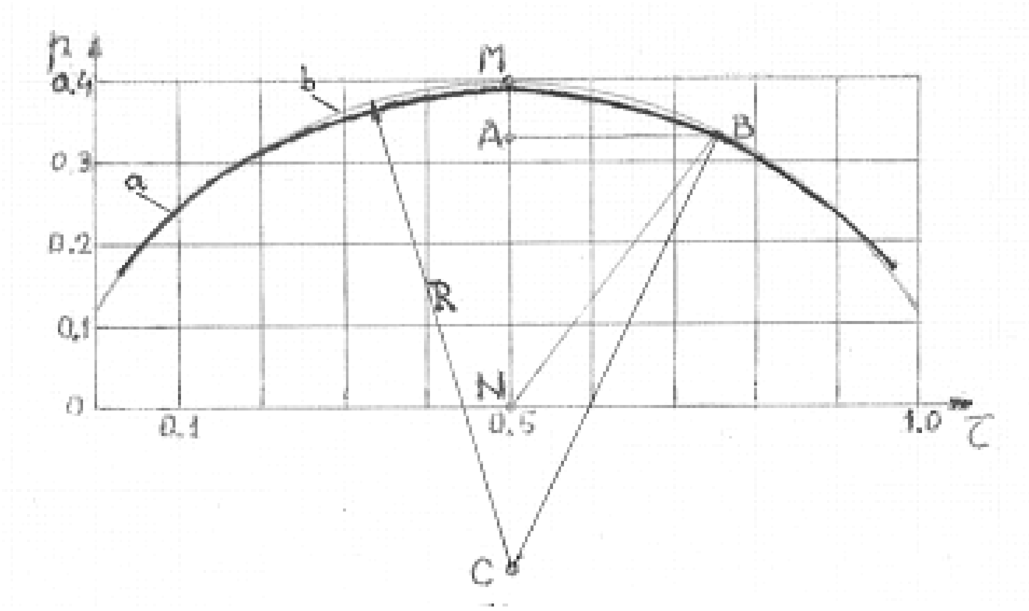

FIGURE 3.

relation (2) becomes:

$$\theta = [1 + (tg\omega)^{0.7}]^{-1/0.7} tg\omega \ . \tag{3}$$

Since the graphical representation of the function δ for $k = 0.7$ makes also the curve "b" distinct from the curve "a", we use the possibility to modify the curve "b" by raising to some power the function θ [6]. Thus we have:

$$\delta = \theta^p \ . \tag{4}$$

Since constant values for the power p did not give useful results and in order to determine the variation of p in function of τ we will use the values of δ from the graph which represent $\delta = f(\tau)$ which is curve "a" of Fig. 2. This is easily done by applying log in the relation (4). We then have:

$$p = \frac{ln\delta}{ln\theta} \tag{5}$$

Doing that we obtain the curve "a" of Fig. 3 which represent the function $p = f(\tau)$.

In order to avoid mistakes in calculations, in Fig. 3 for the τ variable we use a representation scale which bring us to make equivalent the value of $\tau = 1.0$ with 10 cm and for the function p we used similarly as a representation scale which makes equivalent the value $p = 1.0$ with 10 cm. The curve "a" is traced between $\tau = 0.025$ and $\tau = 0.95$.

We see that the curve "a" of Fig. 3 can be assimilated, very close identically with an arc of a circle (curve "b") symmetrical to a vertical axis which poses through $\tau = 0.5$ (the p axis). Thus the center of the respective circle is found on this axis at the point C. For this circle to include in the best manner the arc "a" its radius must have the value $R = 5.9$ cm. Using for R the same geometrical representation scale as for τ and p, we continue to use the value for $R = 0.59$. For $\tau = 0.5$, this circle passes through a point (corresponding with $p_{max} = 0.4$) situated at a very small difference from pmax situated on the curve "a". The relation between p and τ is found using the right angle triangle ABC. Considering the fact that $CN = R - p_{max} = 0.59 - 0.4 = 0.19$ and thus

$AC = 0.19 + p$. Then we can have:

$$R^2 = (\tau - 0.5)^2 + (0.19 + p)^2. \tag{6}$$

Introducing in (6) the value of $R = 0.59$ previously established, we have:

$$p = [0.098 - \tau(\tau - 1)]^{1/2} - 0.19\ . \tag{7}$$

Now we have all the necessary elements to define our Mathematical Model for the Universe Expansion. We recall that this model has the purpose to transpose in an algebraic (and in our case in a trigonometric) form the function $\delta = f(\tau)$ represented in a graphical form using the curve "a" of Fig. 2. The form of the respective Mathematical Model is represented by relation (4) where θ is given in its turn by relation (3) where $\omega = (90\tau)[^\circ]$. The power p of relation (4) is given by relation (7) and is also a function of τ. The form of the equation which represents the variation of δ in function of τ seems very simple at the first glance.

In reality, considering the evidence of all algebraic and trigonometric elements which are comprised in this equation shows that this is a very complex form. This think impose laborious calculations, especially when we desire to use it to determine some measures derived from it, as could be for example, the expansion velocity of the Universe in function of time. The calculations effectuated based on this Mathematical Model bring us to the values of δ (in function of τ) marked with little circles "c" next to the curve "a" in Fig. 2. We find that the curved line which unite the little circles mentioned, is so much close to the curve "a" and practically, it overlaps "a" mostly over the entire variation domain of τ between $\tau = 0.05$ and $\tau = 1.0$. For a small interval situated between $\tau = 0$ and $\tau = 0.05$ the values of $\delta = f(\tau)$ resulted from applying the Mathematical Model defined above are much larger than those resulted from the "Expansion Bell" with as much the values of τ get nearer to $\tau = 0$. Anyway in this domain of values for τ, the velocity of the Universe Expansion is extremely large and this determined that this respective portion of the "bell" to be characterized as "inflation" (see Fig. 1).

3. Conclusions

Presently, the most studied phenomena in Cosmology known under the name of the "Universe Expansion" is graphically represented by a spatial perspective image often seen in the speciality literature [5]. In this image is represented the variation of the "distance" between the principal material and immaterial formations from the Universe regarding the Galaxies as material formations in function of the time measured from the time when the Bing-Bang (*B-B*) was produced. We named this image the "Expansion Bell of the Universe" (*UEB*) and we exposed our point of view in what way it should be regarded in the section 1 from above.

In order to transpose this spatial representation in a Mathematical Model, we imagined a section through (*UEB*) with a plane which contained the time axis. In this way we obtained the graphical representation from Fig. 1. To simplify the reasoning in what followed, we traced the curve "a" of Fig. 2 based on Fig. 1. The coordinate system for this curve is formed from $O\tau$ axis for the abscissa axis for time and $O\delta$ (ordinate axis), for the distance. The measure (no dimensional) τ represents the relation between the current time t and the time developed from the *B-B* moment to the "present", thus $t_{max} = 13.7$ billion years. The measure (no dimensional) δ represents the relation between the distance l, at the moment t and the maximum distance from present, thus $l_{max} = 93$ billion light-years. The curve "a" of Fig. 2 was compared with the curve which represents the paratrigonometric function "sin paratrigonometric of order k of the angle (90τ) i.e. $spr_k(90\tau)$" studied in [6]. By trying we get the conclusion that the most agreeable value for our case is $k = 0.7$ (curve "b" of Fig. 2), but in addition it was necessary to raise to a certain power p this respective function. We determined (established) using a graphical analysis the function $p = f(\tau)$. In this way we established the needed Mathematical Model for the curve "a" of Fig. 2. This is represented by the relation (4) where θ in its turn is given by the relation (3) and $\omega = (90\tau)[^\circ]$. The power p is given by relation (7). This Mathematical Model is graphically marked by small circles very closed with the curve "a" of Fig. 2 in the domain of values $\tau = 0.05$ to 1.0 . In other words, the established Mathematical Model covers in a proportion of 95% the domain of the values which the variable τ can have.

Moreover it is worth to mention the fact that this Mathematical Model covers the entire domain of the time in which the Universe is manifesting under material form (stars, planets, galaxies) as we can see in Fig. 1.

References

[1] Hawking, S.W., *The Theory of Everything. The Origin and Fate of the Universe*, Phoenix Books and Audio, "Humanitas" Publishing House, Bucharest, 2014 (published in Romanian)

[2] Sing, S., *The Origin of the Universe*, "Humanitas" Publishing House, Bucharest, 2008 (in Romanian)

[3] Moreno, R., Destua, S., Rosa, M., Garcia, B., Universe Expansion, http://sac.csic.es/astrosecundaria/ro/cursos/formato/materiales/ppts/talleres/T8_ro.pdf, (in Romanian)

[4] https://ro.wikipedia.org/wiki/Big_Bang

[5] Lazar, A., *Hubble Constant. Universe Evolution (4)*, Contact Sciencia (/contact-sciencia.html), (in Romanian)

[6] Baica, M., Cardu, M., *The Paratrigonometry, Nonconventional Trigonometry*, A.G.I.R. Publishing House, Bucharest, 2010

The University of Wisconsin
Department of Mathematics
Whitewater, Wisconsin, 53190, U.S.A.
E-mail address: baicam@uww.edu

S. C. Herbing S.R.L.
Str. Otetari, 9, Sector 2, Bucharest, Romania
E-mail address: mircea.cardu@herbing.ro