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REGARDING THE STABILITY OF ROTORS RUNNING IN VARIABLE STIFFNESS BEARINGS

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Abstract. Low bearings stiffness gives rise to the rotor steady state response, this state being superior to that obtained with moderate and high bearing stiffness. The paper describes a combined theoretical-experimental investigation into the stability of a flexible rotor running in flexible bearings, with particular references to low stiffness bearings. Results show that the bearings type proposed can be used effectively without resulting in system instability.

1. Introduction

Many rotors operate at speeds which are in excess of one or more critical speed values. So, this fact creates a problem for the designers who must take into account that the system stability has to be satisfactory in terms of covering the entire range of operating speeds.

Generally, the features illustrating the stability of the rotor-bearings depend on the dynamic stiffness of the rotor bearings and on that one of the foundation on which the bearings are mounted. For operation at speeds less than the third critical speed, the specialized literature mentions that bearings whose stiffness is very low give a superior performance to others so far as steady state operation is concerned. Although, conventional bearings capable of carrying heavy loads are provided with a lubricant film characterized by significant stiffness values. For these reasons, specialists are seeking to identify new alternative for rotor boundary.

The proposed solution was the magnetic bearing whose dynamic stiffness can be easily modified by the machine operator. Several disadvantages (high price, operation lack of security) limited the use of this constructive type.

Another proposed alternative was to use a special oil film bearing which allows system stable state operation under heavy loads and whose stiffness can be tuned to the levels required. Such a support type can be achieved either in a hydrostatic mode, or as a controllable squeeze lubricant film damper. The controllable squeeze lubricant film damper bearing type is shown in Figure 1. Hydrostatic action of lubricant film pressurized bearing channels centralizes the shaft in the bearing, offsetting any static loads applied to bearing. In dynamic regime, however the bearing stiffness may be reduced to a very low value, by the accumulators connected to the bearing recesses via remote controlled valves. When the valves are fully opened, the lubricant film circulation is not obstructed, and so the dynamic bearing stiffness decreases significantly.

2. Theoretical considerations

Forces F and moments M acting within a system consisting in an easily flexible shaft supporting three rotors are presented in figure 2.

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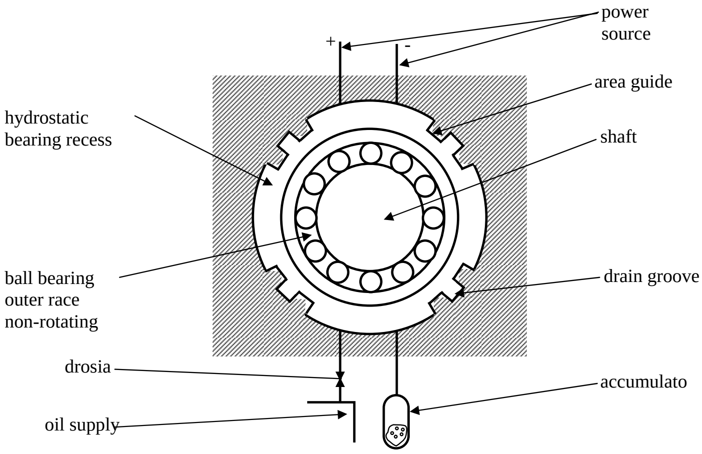

FIGURE 1. Hydrostatic squeeze-film bearing diagram

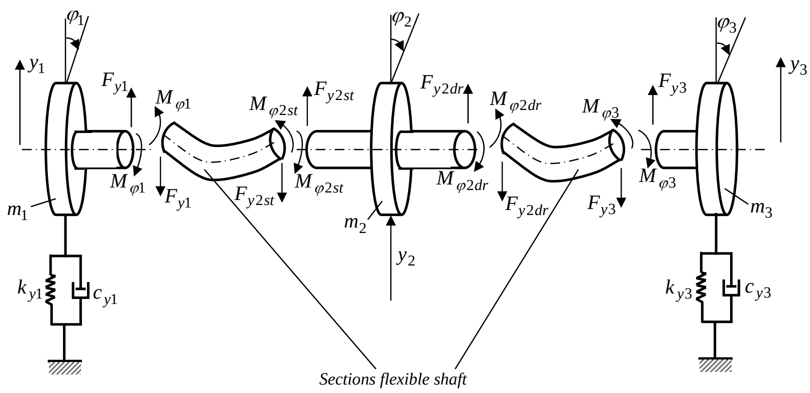

FIGURE 2. General arrangement of the rotor and shaft sections

The motion equations for the first rotor can be written as:

$$\left.\begin{array}{ll} m_1 \cdot \ddot{x}_1 + c_{x1} \cdot \dot{x}_1 + k_{x1} \cdot x_1 = F_{x1} & a) \\ m_1 \cdot \ddot{y}_1 + c_{y1} \cdot \dot{y}_1 + k_{y1} \cdot y_1 = F_{y1} & b) \\ M_{\theta 1} = J_1 \cdot \ddot{\theta}_1 & c) \\ M_{\phi 1} = J_1 \cdot \ddot{\phi}_1 & d) \end{array}\right\} \quad (1)$$

where m_1 is the mass of the first rotor, and J_1 its moment of inertia toward the axis of rotation. Also, k_{x1} and k_{y1} are the rotor stiffness coefficients on the horizontal direction x, respectively vertical one y, and c_{x1}, c_{y1} are the damping coefficients on these directions. Rotor angular displacement on the vertical axis is θ, and on the horizontal axis is φ.

Similarly, the equations of motion for the second rotor are:

$$\left.\begin{array}{ll} m_2 \cdot \ddot{x}_2 = F_{x2st} + F_{x2dr} & a) \\ m_2 \cdot \ddot{y}_2 = F_{y2st} + F_{y2dr} & b) \\ M_{\theta 2st} + M_{\theta 2dr} = J_2 \cdot \ddot{\theta}_2 & c) \\ M_{\phi 2st} + M_{\phi 2dr} = J_2 \cdot \ddot{\phi}_2 & d) \end{array}\right\} \quad (2)$$

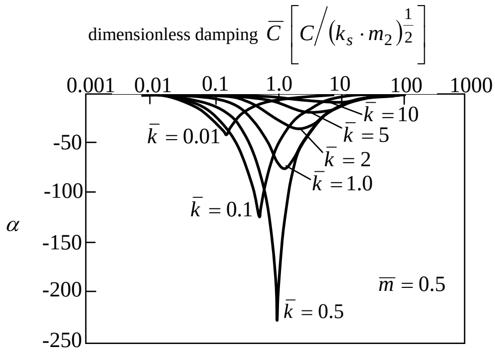

FIGURE 3. Variation of stability parameter with bearing damping

For the third rotor the motion equations are similar to those written for the first.

Applying the beam deflection theory, a relationship between forces, moments and displacements for sections 1-2 and 2-3 of shaft can be established, both in the xz plan and the yz plan. This matrix can be written [1] as:

$$[M] \cdot u^2 \{X\} + [C] \cdot u \{X\} + [K] \{X\} = 0 \tag{3}$$

where square matrices are of magnitude 12×12 and $\{X\}$ is a column matrix indicating all 12 degrees of freedom $x_i,\ y_i,\ \theta_i,\ \varphi_{i(i=1,2,3)}$ for each rotor (matrix $[M]$ is the inertia matrix, matrix $[C]$ is the damping matrix, and $[K]$ is called the stiffness matrix).

Equation (3) can be re-written as:

$$\begin{bmatrix} 0 & [M] \\ [M] & [C] \end{bmatrix} \begin{Bmatrix} u^2 \{X\} \\ u \{X\} \end{Bmatrix} + \begin{bmatrix} -[M] & 0 \\ 0 & [K] \end{bmatrix} \begin{Bmatrix} u \{X\} \\ \{X\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{4}$$

can be abbreviated to:

$$\left[\bar{M}\right] \cdot u [D] + \left[\bar{K}\right] \cdot [D] = [0] \tag{5}$$

$\left[\bar{M}\right]$ and $\left[\bar{K}\right]$ being the inertia and stiffness matrices of dynamic oscillating system.

Equation (5) is an eigenvalues problem which was solved on the computer to determine the displacement matrix $[D]$ content and the corresponding values of u – the eigenvalue system. Generally, u is of $u = \alpha + i\beta$ form, where β is the oscillation frequency, and α is the stability parameter. This parameter is measuring the transient oscillations decrease frequency, representing the largest real part of the eigenvalue system, u.

3. COMMENTS REGARDING THE THEORETICAL RESULTS OBTAINED

The theory described throughout Chapter 2 was transposed into a computer programme used to obtain results for different parametrical values of the oscillating system. In each investigation the largest value of α among all roots u of the equation (5) was noted. The eigenvalues corresponding to the shaft vibration modes were analyzed and graphics were drawn.

Figure 3 shows the variation of α with the bearing damping for an oscillating system whose report masse ratio (end rotor mass m_1/ midspan rotor mass m_2) is $0,5$. Curves are presented for systems with different bearing/shaft stiffness ratios (k_b) / (k_s). Graphics obtained indicate the optimum damping value (for the smallest value ofα) necessary to ensure the maximum stability.

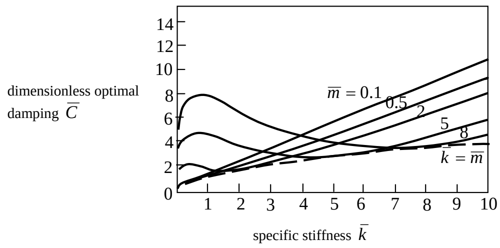

FIGURE 4. Variation of optimal damping bearing with specific stiffness

The upper figure shows the situation in which the system is stabile and the variation of the parameter α in the negative range of values. In case α is minimum (α) the system is maximum stabile. Figure 3 shows that, for any oscillating system with given specific mass and stiffness, there is a particular bearing damping coefficient value for which the stability is maximum, depending clearly on the system specific rigidity.

Figure 4 indicates the optimal damping value having different specific weights ($\bar{m} = m_1/m_2$). In each situation the optimal damping variation depends on the effective stiffness $\bar{k}$ ($\bar{k} = k_l/k_r$). The figure shows how the value of optimum bear damping coefficient, for maximizing stability, varies with system stiffness ratio and mass ratio.

4. Conclusions

The authors aim to diversify their research in two main directions: to theoretically identify the influences that a possible use of some coupling transmitting motion from one shaft to another would have on the stability of the rotor-bearings; to do measurements in situ on a rotor-bearings system so as the experimental results validate the theoretical research.

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