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DUAL AND BIPOLAR MATHEMATICAL MODELS DERIVED FROM THE SOLUTIONS OF TWO VERY SIMPLE MATHEMATICAL EQUATIONS AND THEIR APPLICATION IN PHYSICS

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Abstract. In this paper we develop a "Dual" model for the use in for solving wave and corpuscle problems in Quantum Mechanics relative to behavior of light in some diverse processes in Physics. For example, in the course of emissions and absorption in the Photo Electric Effect and in the Compton Light Effect there are observations of a corpuscular nature. Conversely, while observing the phenomena of interference, diffraction, interference and polarization, light exhibits an oscillatory character. The dual model will, applied to these phenomena, be able to distinguish among these separate effects. We further perform a "Bipolar" model for problems common to electric fields and the magnetic fields (these fields must occur simultaneously). Each propagation direction of the electromagnetic wave can be represented by two sinusoid curves situated in two perpendicular planes. One of them represents the oscillations of the electrical intensity vector and the other represents the oscillations of magnetic intensity vector. These can be accorded a unified treatment by the proposed new bipolar model. There are many such phenomena in Physics which exhibit either a dual behavior or a bipolar behavior. The work is to be based upon algebra, mathematical logic, quantum theory and electromagnetic theory. Both models will emerge from the solutions of similar but not identical equations.

1. Introduction

We start analyzing the properties of the roots of the equation $y^2 - 1 = 0$ in conditions of the actual (conventional) definition of the square root of a number and of "the connector" OR from the Mathematical Logic. Also, we analyze the properties of the equation $y^2 + 1 = 0$ in conditions of the authors proposed alternative definition of the square root of a number and of "the connector" AND from the Mathematical Logic. We will put in evidence the "selective" properties of the roots of the above mentioned equations in relation with two terms of an algebraic sum, by introducing them together in the specific mathematical relations. Based on that, the respective roots are applied to formulate a unique mathematical model for some phenomena in Physics which presently are represented by two different models.
Note: We explain this in detail not to insult the knowledge of the readers but to make an easy transition to the Mathematical Logic concept.

Consider the two equations

$$y^2 \pm 1 = 0 \tag{1}$$

Let analyze the equations in (1) using the classical (conventional) definition of a square root of a number formulated as "a number which multiplied with itself reproduce the given number". We mention that the square root defined as above is a number considered algebraically including its algebraic sign + (plus) and respectively − (minus). In order to distinguish the situations derived from the equations (1), instead of y we use the letters

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h, i (known to be $\sqrt{-1}$) and j. For the beginning let analyze the following equation from (1)

$$h^2 - 1 = 0 \tag{2}$$

(2) can be written as

$$h^2 = 1 \tag{3}$$

The two roots of the above quadratic equation (3) are

$$h_{1,2} = \sqrt{1} = \pm 1 \tag{4}$$

The relation (4) shows that that a positive number (in our case 1) has two quadratic roots with the same absolute value, but they are different by algebraic sign (+ or $-$).

If we separate the two roots in the relation (4) we use the connector OR (with the symbol $\vee$) of Mathematical Logic [1] we can write

$$h_1 = +1 \ \vee \ h_2 = -1 \tag{5}$$

It is true that both $(+1) \cdot (+1) = 1$ and $(-1) \cdot (-1) = 1$.

The connector OR marks the fact that each of the two roots are independent from each other.

Because the two roots from relation (5) have the same absolute value $|1|$, we name them “dual unitary roots” since they are tied together with the connector OR. Simply, we will summarize the relation (5) by the notion “Dual Units”.

Along with the equation (2) of equation (1) we also derive the equation

$$i^2 + 1 = 0 \tag{6}$$

The equation (6) does not have real roots and thus i can’t be expressed in the other ways then

$$i = \sqrt{-1} \tag{7}$$

Thus we evidently have

$$i^2 = \left(\sqrt{-1}\right) \cdot \left(\sqrt{-1}\right) \tag{8}$$

We know that the imaginary number $i = \sqrt{-1}$ is named “Imaginary unit” [2]. It has a wide utilization consecrated in the domain of the complex numbers.

2. Quadratic bipolar roots

Our further exposure is based on a non-conventional alternative of the classical definition of a root square of a number introduced at the beginning of the previous chapter. Thus we propose the following definition of a square root of a number used only for the negative number case and as we will further see, it is valid only if it is used in the case if we tie them with the connector AND (symbolized with $\wedge$) from the mathematical logic:

“The square roots of a negative number are two numbers whose absolute values are equal and multiplied give the absolute value of the given number, but the algebraic signs of the two roots are different (+ and respectively $-$). Defined like this and using the connector AND, as we have shown above, an equation of type (6) in which we replaced i by j will have the roots:

$$j_1 = +1 \text{ and } j_2 = -1 \tag{9}$$

Using these roots for the reverse calculations we obtain:

$$j^2 = (+1) \cdot (-1) = -1 \tag{10}$$

The relation (10), valid in the case of the application of the unconventional definition for the square root of a negative number, is very much alike with the relation (6) which is tied by the classical definition of the square root of a negative number.

The essence of this reasoning in this chapter is made by the connector $\wedge$, which tie the two roots j_1 and j_2. By this connector the respective roots are inseparable tied together. Thus, none of the two roots can "exist" without the other. In other words $j_1 = +1$ can't "exist" unless is together with $j_2 = -1$ and vice versa. For this reason we named them "bipolar roots" or simpler "bipolar units". These two roots ($j_1 = +1 \ \wedge \ j_2 = -1$) are real roots unlike the unique root $i = \sqrt{-1}$ which having the same origin it still remain imaginary. We could say that obtaining the real character of the roots ($j_1 = +1 \ \wedge \ j_2 = -1$) was "paid" by losing their "freedom".

3. The selective character of the Dual Units h and Bipolar Units j

As we will see in this chapter, there exist situations when the same physical phenomena manifested in two aspects are represented by two distinct mathematical models. In what follows, we will show that the dual units h (abbreviated DUh) and the bipolar units j (abbreviated BUj) have the properties to select in function of the necessity, one or the other of the two mathematical models mentioned, from an unique mathematical model which is characteristic to the refereed physical phenomena. In order to apply them selectively we denote these two mentioned above mathematical models DUh and BUj with M and N respectively and they will become terms in a unique formula as we will see below.

This unique relation in which both terms M and N are present is the following:

$$F_{1,2} = \left(\frac{1}{2}\right) (h_{1,2} + 1) \ (M + N) \ - h_{1,2}N \tag{11}$$

If we introduce the connector $\vee$ [see the relation (5)] in the relation (11), this will become:

$$F_{1,2} = \left(\frac{1}{2}\right) [(h_1 \vee h_2) + 1] \ (M + N) \ - (h_1 \vee h_2) \cdot N \tag{12}$$

For $h = +1$ we have $F = M$ or ($\vee$) for $h = -1$ we will have $F = N$. In other words the relation (12) can be written as:

$$F_1 \vee F_2 = M \vee N \tag{13}$$

Based on the relation (12) we can imagine another one which will have on the left side of the equality two terms $A; B$ connected by the sign $+$ (plus). For example it is logic to symbolize by $A; B$ some unique measures which symbolize synthetically the character of a phenomena in Physics and by $M; N$ to represent mathematical relations between other characteristic measures which intervene in that phenomena. A relation on which we can make a selection to get

$$A = M \vee B = N \tag{14}$$

Is the following:

$$\left\{ \begin{array}{l} \left(\frac{1}{2}\right) [(h_1 \vee h_2) + 1] \ (A + B) \ - (h_1 \vee h_2) \cdot B = \\ \left(\frac{1}{2}\right) [(h_1 \vee h_2) + 1] \ (M + N) \ - (h_1 \vee h_2) \cdot N \end{array} \right\} \tag{15}$$

Introducing in (15) one after the other the values of the dual roots $h_1; h_2$ of (5) we get the relation (14). With this relation we put in evidence the selective character of DUh. We have to mention that in the relations (11), (12) and (15) above we used the algebraic signs $+$ (plus) and $-$ (minus) only to be able to operate in these relations using algebraic rules. In the case when we refer to the measures in Physics the operations of adding and subtracting which intervene in the respective relations are not homogenous from the point of view of the measure units, as we will see in the next chapter. Also, the commutative character of the sums $(A+B)$ and $(M+N)$ is not valid in our case, since if it is applied to only one of these sums it will alter the validity of the relation (14).

Similar with the model of applying selectively DUh developed above, we will use for the same purpose a methodology for BUj, where in (15) the operator $\wedge$ instead of the operator $\vee$ is used and we can write the following relation:

$$\left\{ \begin{array}{l} \left(\frac{1}{2}\right) [(j_1 \wedge j_2) + 1] \ (C + D) \ - (j_1 \wedge j_2) \cdot D = \\ \left(\frac{1}{2}\right) [(j_1 \wedge j_2) + 1] \ (P + Q) \ - (j_1 \wedge j_2) \cdot Q \end{array} \right\} \tag{16}$$

Using the relation (9) we obtain the selection relation

$$C = P \wedge D = Q \tag{17}$$

This is very much alike with the relation (14). Surely, regarding what we had shown before regarding the use of the algebraic signs + (plus) and − (minus) are valid in this case, also.

In the following chapter we will discuss som practical applications of DUh and BUj.

4. The applications in Physics of the Dual Units h and of the Bipolar Units j

4.1. **Duality case Corpuscle - Wave characteristic of the light.** One of the important chapters in the Quantum Mechanics is that one regarding the duality corpuscle – wave. This is referring to the various behavior of the light in diverse physical processes. Thus, in the processes of emission and absorption, in the photo electric effect and in the Compton light effect there are observations of a corpuscular nature. Conversely, while observing the phenomena of interference, diffraction and polarization, light has an oscillatory character.

In this case the physical entry is unique: the light. Light expression in function of the conditions required to behave have a dual character. Evidently this is a typical case where we can apply the selective capacity of the Dual Units h.

It is known that the corpuscular behavior of the light, in fact of the photon, appears in evidence two of its characteristics namely the energy E and the impulse p [3, 4].

Between these two measures this relation exists

$$E = p \cdot c \tag{18}$$

where, c is the velocity of light.

On the other way, for the oscillatory manifestation of the light there are characteristic the frequency ν and the wave length λ. The relation between these two measures is:

$$\lambda = \frac{c}{\nu} \tag{19}$$

Using the model of the relation (15), the relations (18) and (19) can be included in the following unique relation.

$$\left\{ \begin{array}{l} \left(\frac{1}{2}\right) [(h_1 \vee h_2) + 1] \ (E + \lambda) \ - (h_1 \vee h_2) \cdot \lambda = \\ \left(\frac{1}{2}\right) [(h_1 \vee h_2) + 1] \ \left(pc + \frac{c}{\nu}\right) \ - (h_1 \vee h_2) \cdot \left(\frac{c}{\nu}\right) \end{array} \right\} \tag{20}$$

The selective character of DUh is expressed by the fact that for $h = +1$, the relation (20) becomes (18), and for $h = -1$, the relation (20) becomes (19).

Thus, the duality corpuscle – waive which characterize the light can be mathematically represented either by the relation

$$E \vee p \tag{21}$$

which represents the corpuscle character of the light or by the relation

$$p \cdot c \vee \frac{c}{\nu} \tag{22}$$

which represents the oscillatory character of the light.

4.2. **The case of the electromagnetic waves.** It is known that the electromagnetic waves are phenomena in Physics which consist in an electric field and in a magnetic field both in the same space which generate one another in function of their propagation.

For each propagation direction the electromagnetic wave can be represented by two sinusoidal curves found in two perpendicular planes of each other. One of them represents the oscillations of the "Electrical Intensity" vector $\vec{E}$, and the second represents the oscillations of the "Magnetic Intensity" $\vec{H}$. The two vectors oscillate at the same phase. If we admit that the two waves are propagated in the direction of Oy, their equations are:

$$\vec{E} = \vec{E}_0 \cos w \left(t - \frac{y}{v}\right) \tag{23}$$

$$\vec{H} = \vec{H}_0 \cos w \left(t - \frac{y}{v}\right) \tag{24}$$

where $\vec{E}_0$ and $\vec{H}_0$ represent the amplitude of the electric intensity vector and respectively of the magnetic intensity vector. The actual time is denoted by t and y is the actual coordinate on the Oy axis. The wave propagation velocity v is given, in its turn, by the relation

$$V = \frac{\lambda}{T} \tag{25}$$

where λ is the wave length of the considered oscillation and T is its period. In its turn, the period T is given by the relation

$$T = \frac{2\pi}{w} \tag{26}$$

where w is the frequency of the oscillation.

The phenomena of the electromagnetic waves represent a typical application of the Bipolar Units (BUj). It is true, the two fields, electric and electromagnetic exist only together and we can consider that the relation between them can be characterized with the corrector AND ($\wedge$).

On the other side, these two components of the electromagnetic waves are modeled mathematically using specific measures as in relations (23) and (24). In this way, using the model of the relation (16), we unify the relations (23) and (24) in the following relation:

$$\left\{ \begin{array}{l} \left(\frac{1}{2}\right) [(j_1 \wedge j_2) + 1] \left(\vec{E} + \vec{H}\right) = \\ \left(\frac{1}{2}\right) [(j_1 \wedge j_2) + 1] \left[\vec{E}_0 \cos w \left(t - \frac{y}{v}\right) + \vec{H}_0 \cos w \left(t - \frac{y}{v}\right)\right] \end{array} \right\} \tag{27}$$

For $j_1 = +1$, the relation (27) becomes (23) and for $j_2 = -1$ relation (27) becomes (24).

The bipolar character of the light, given by the simultaneous existence of the two oscillation vectors "Electrical intensity" and "Magnetic intensity", is marked by the corrector $\wedge$ which intervene in the relations:

$$\vec{E} \wedge \vec{H} \tag{28}$$

$$\vec{E}_0 \cos w \left(t - \frac{y}{v}\right) \wedge \vec{H}_0 \cos w \left(t - \frac{y}{v}\right). \tag{29}$$

5. Conclusions

Considering the discussion in the previous chapters we can enumerate some important conclusions:

5.1. The equation $y^2 - 1 = 0$ has two roots denoted $h_1 = +1$; $h_2 = -1$ and we named them as "Dual Units" (DUh). If we appeal to the connector OR (symbolized by $\vee$) from the "Mathematical Logic" the two roots being independent, the relation between them can be written $(h_1 \vee h_2)$, respectively $(+1 \vee -1)$.

5.2. If we apply the classical definition of the root square of a number, then one of the roots of the equation $y^2 + 1 = 0$ is $i = \sqrt{-1}$, which we know that is named "Imaginary unit" and is used in Mathematics specifically in Complex Analysis.

In this paper we proposed for application one variant of the classical definition of a root square of a number. Conform this definition the root square of a number (roots of which product is equal with the given number) are equal as absolute value only, there algebraic signs being opposite, + (plus) and − (minus) respectively.

In this case, the roots of the equation $y^2 + 1 = 0$ are $j_1 = 1$; $j_2 = -1$. The connection between these two roots is done by the connector AND (symbolized by $\wedge$), what means that they are not independent and they are always together.

5.3. The dual units (DUh), that is $(h_1 = +1) \vee (h_2 = -1)$ have the capability to select one of the terms M, N respectively which are put together in a relation of the form $(M + N)$. Because of this we use a special mathematical relation in which the two roots $(h_1 \vee h_2)$ and M; N appear. The two separated terms keep between them a relation characterized by the connector OR ($\vee$), the same as the relation between the two roots $(h_1 \vee h_2)$. In other words saying, in this case, the terms M; N can't "exist" together but in a situation characterized by the preposition "or M, or N".

The bipolar units (PUj), that is $(j_1 = +1)(j_2 = -1)$ have the same capability to select as DUh, with respect to the two terms P; Q. Also, these two terms put together under the form $(P + Q)$, can be separated by a special mathematical relation. But in this case, it results that the relation between the terms P; Q, now separated is characterized by the connector AND ($\wedge$). This means that the respective terms can "exist" only together. A such situation is characterized by the preposition "and P and Q".

5.4. The selection capacities of DUh are suited very well to be used in making evident from a unique mathematical relation for a term which represent one or other different behavior which a phenomena can have, in function of which this can manifest itself. An example of such kind discussed in this paper was "The duality corpuscle – wave".

Also, the selection capacities of BUj can be used in making evident from a unique mathematical relation for a term which represents one or other component for a phenomena in Physics where these components are physical inseparable. The example for this case analyzed in this paper are "The electromagnetic waves".

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