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BAICA-CARDU PARATRIGONOMETRY, A GENERALIZATION OF THE CLASSICAL AND SOME NEW NON-CLASSICAL TRIGONOMETRIES AND ITS APPLICATION IN MECHANICS AND WAVE THEORY

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Abstract. In their previous papers the authors introduced some new Trigonometries as: 1. Quadratic Trigonometry (QT), 2. Polygonal Trigonometry (PT), 3. Trans Trigonometry (TT), 4. Infra Trigonometry (IT), 5. Ultra-Trigonometry (UT), 6. Extra Trigonometry (ET), 7. Para-Trigonometry (PRT). This time in this paper we perform a synthesis of all these Trigonometries and state some of their applications.

1. Introduction

It is known that in the most technical domains and in the wave theory the oscillation character predominates and as such, in many cases these phenomena can be mathematically modeled using the trigonometric functions *sina* and *cosa*, respectively. Of course, there are many oscillation phenomena of which mathematical representation does not have a sinusoidal form and in their analysis using the Classical Trigonometry is useless.

In this case the decomposition of these functions in Fourier series is used in order to do the mathematical modeling needed. Therefore starting with this difficulty the authors felt the need to invent new non-classical trigonometries where the basic trigonometric figure specific to each individual non-classical trigonometry is going to overcome the above mentioned oscillation problem.

2. Authors previous results

It al started with the well known relation in the Classical Trigonometry (CT)

$$\sin^2\alpha+\cos^2\alpha=1 \tag{1}$$

which for QT becomes

$$|sq\alpha|+|cq\alpha|=1 \tag{2}$$

for (PT) is

$$sp^k\alpha+cp^k\alpha=1 \tag{3}$$

for (TT) is

$$|st_k\alpha|^k+|ct_k\alpha|^k=1 \tag{4}$$

for (IT) is

$$|si_k\alpha|^k+|ci_k\alpha|^k=1 \tag{5}$$

for (UT) is

$$|su_k\alpha|^k+|cu_k\alpha|^k=1 \tag{6}$$

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for (ET) is

$$|se_k\alpha|^k + |ce_k\alpha|^k = 1 \tag{7}$$

and for (PRT) is

$$|spr_k\alpha|^k + |cpr_k\alpha|^k = 1 \tag{8}$$

The Para-Trigonometry comprises all of the others Trigonometries and in the following section we will give a summary of this fact.

The inclusions table of all the Classical and Non-Classical Trigonometries in the Para-Trigonometry. The key of this discussion is the order of k in the relations of Section 2.

The name of thePRT structure	The value domain of the order k	The basic trigonometric figure form (BTF)	The category as part of PRT
Infra- -Trigonometry (IT) 4.	$0 \leq k < 1$	Rhombus with curved sides and the concavity opposite to the reference O*	Infra- -Trigonometry (IT) 4.
Quadratic Trigonometry (QT) 1.	$k = 1$	Rhombuses with straight sides (rhombus with "para-trigonometric mirrors")	Quadratic Trigonometry (QT) 1.
Trans- -Trigonometry (TT) 3.	$1 < k < 2$	Rhombuses with curved sides and the concavity towards the reference O*	Extra- -Trigonometry $1 < k \leq \infty$ (ET) 6.
Classical Trigonometry	$k = 2$	Trigonometric circle	
Ultra- -Trigonometry (UT) 5.	$2 < k \leq \infty$	Rhombuses with curved sides and the concavity towards the reference O*	

*) The coordinate system origin Ox-Oy

**) Polygonal -Trigonometry (PT)-2) is referring to the domain $1 < k < 2$, but k is variable in function of the number of the sides in the trigonometric polygon.

All of the above trigonometries in the table are contained in the Para-Trigonometry (PRT) where k is less or equal with zero and greater or equal with infinity.

3. Applications of the Para-Trigonometry

The toothed wheels are very often used in the domain of the machines and mechanical installations in the diverse transmission systems with chain, conveyers, elevators, the rolling systems with caterpillars and especially gearings. In function of their utilization the teeth profile have diverse geometrical forms.

The polar coordinates can be applied to define a complete denture of a toothed wheel mathematically with the Para trigonometric function named "Para trigonometric sinus" for four values of the Para trigonometric order k.

There are many graphs, which can be used in technology and they are obtained from giving to the order k some specific values. One can use these results of the Para Trigonometry as they please in modeling mathematically many applications problems.

4. Conclusions

(i) The generic denomination of all these trigonometries including the Classical-Trigonometry is the PARA-Trigonometry (PRT), where $0 \leq k \leq \infty$. In another words, all these trigonometries can be comprised in the notion of the Para-Trigonometry.

(*ii*) The basic relations and all the others from [6] can be applied in the case of all above mentioned trigonometries, distinguishing themselves by the domain of the values for the "order" k.

(*iii*) The authors also believe that some computer programming can be developed to get some graphs of the curves in function of the order k.

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